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Military System Reliability: Department of Defense Contributions

J. SPIEGEL† AND E.M. BENNETT†

Summary—This report describes the Defense)epartment's increasing concern regarding elec- ronic equipment reliability during the period 942-1959. It discusses the establishment of the oint Army-Navy Vacuum Tube Development Com- nittee (VTDC) in June, 1943, and VTDC's suc- :essor, the Panel on Electron Tubes (PET) in)ctober, 1946. Also described is the formation of he Ad Hoc Group on Reliability of Electronic :quipment in December, 1950, the Advisory Group n Reliability of Electronic Equipment (AGREE) in \ugust, 1952, the Advisory Group on Electron Tubes (AGET) in March, 1954, the Advisory Group n Electron Parts (AGEP) in June, 1954, and the \d Hoc Committee for Guided Missile Reliability ACGMR) in March, 1957. The interrelation of the asks and findings of these organizations are dis- :ussed.

INTRODUCTION

The achievement of adequate reliability in any complex system involves appreciable technical and managerial innovation along with particularly nigh standards of production and use. This is particularly the case when the system is a mili- ary one and when it is also highly dependent upon electronics. Therefore, it is not surprising to note that the Department of Defense has played a vital role in determining the direction of a variety of reliability efforts, especially in the areas of military electronics.

1942—1949

The modern quest of the Department of Defense for higher reliability in military electronic sys- tems and equipment started, for all practical pur- poses, with the onset of World War II.[1]

Prior to that war, military electronic equipment was relatively simple, and, since simple electronic devices are more reliable than complex ones, the problems attendant upon unreliability were not greatly stressed. However, concomitant with the onset of our preparations for war came a tremen- dous demand from the military services for a host of what were then seen as relatively complex elec- tronic equipments. New military demands required developmental efforts for new tubes, new circuitry, new applications of components, and an increased ability for the equipment to withstand higher levels of environmental stress.

Since numbers of organizations were interested in developing new tubes and novel applications of tubes, it was realized by many that, without con- trol and coordination, millions of dollars could be spent in this direction without firm results to show for such expenditures. Recognizing this need, members of the Radiation Laboratory of the Massa- chusetts Institute of Technology proposed that a committee be formed to coordinate all such tube developmental efforts.

On November 4, 1942, a Radiation Laboratory report was submitted to the Navy Bureau of Ships. First, noting that the complexity of the vacuum tube is such that it must be considered a separate and distinct piece of apparatus, the report sug- gested that ". . . large and continuing demands for new applications, together with relatively short life and coincident problems of supply, create an unusual volume of problems demanding the exist- ence of a group whose responsibility is confined to vacuum tube coordination." Further, ". . . al- though several of these problems might be referred to existing bodies, none has the authority to control or coordinate the whole subject in any given in- stance nor are these examples inclusive of the scope of the total problem. In the United States there are industrial, Army, Navy, and university

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[1]In this review we have limited our attention to one ap- parently significant flow of historical forces relevant to military system reliability. We have not discussed the multiple contributions either of allied agencies such as inservice military laboratories, of allied fields such as quality control statistics, of allied industrial and engi- neering firms, or of professional groups and organiza- tions. These have not been neglected, however, for any search of the literature in the field of system reliability will quickly demonstrate that many advances are a di- rect result of the efforts of such groups.

vacuum tube laboratories. Within the limits of informality and lack of authority there is a degree of cooperation among these various institutions, but, without direction, duplication, conflict and misguided effort must of necessity occur."

The report went on to detail its specific recommendations for the formation of this proposed committee. It suggested that the committee be organized as an executive body of the Joint Communications Board (JCB) and that its responsibilities should be broad enough to fully coordinate and encourage tube development work.[2]

The Bureau of Ships agreed in principle with the establishment of a coordinating body and, on January 18, 1943, initially recommended that it be made part of the Radiation Laboratory. However, additional consultations between the Chief of Engineering and Technical Service of the Office of the Chief Signal Officer and the Head of the Radio Division of the Bureau of Ships resulted in a proposal for a joint Army-Navy Vacuum Tube Development Committee to be formed by the Office of Scientific Research and Development (OSRD). This final proposal was reviewed by the Joint Committee on New Weapons and Equipment of the Joint Chiefs of Staff and, as a result, the Chairman of the National Defense Research Committee (NDRC) was asked to form such a committee.

On June 7, 1943, the first meeting was held, with Dr. I. I. Rabi as Chairman. Dr. Vannevar Bush, then Director of OSRD, attended the first meeting and expressed his feeling that the initial formation of the Vacuum Tube Development Committee (VTDC) was of an experimental nature and that in the future, if events warranted, it might be taken from NDRC and placed under joint Army, Navy and OSRD sponsorship.

On August 23, 1943, the Director of the Vacuum Tube Development Committee issued the first of what proved to be a series of publications on VTDC history, directives and information. Essentially, these publications were a review of the history of the VTDC and provided information as to the membership of the committee. Included in these reports were restatements of the VTDC responsibilities as well as a detailing of the methods by which the VTDC would live up to them.

Some of the responsibilities were:

1) to keep itself informed regarding vacuum tube research and development;
2) to formulate plans and make recommendations regarding specific vacuum tube research and development programs;
3) to consider problems of vacuum tube research and development suggested by any of its members or by liaison representatives to the committee;
4) to investigate and designate operating conditions for new tubes and to specify tests to determine the suitability for new types of service use;
5) to recommend to the JAN-1 Specifications Committee additions to the joint Army-Navy preferred list of vacuum tubes with preliminary specification data on newly developed tubes;
6) to recommend appropriate action to the Joint Vacuum Tube Control Committee regarding the procurement and assignment of priorities and precedents for the procurement of developmental tubes; and
7) to pass information from one commercial concern to another only with the consent of the originator.

With the VTDC in need of technical facilities and personnel in order to realize its responsibilities, a contract was established between OSRD and Columbia University, under which the Vacuum Tube Development Group was organized, at Columbia, to serve as a Secretariat for the full committee.

Throughout the war, both the Vacuum Tube Development Committee and Group were successful in coordinating efforts in the development of vacuum tubes. A survey of all the vacuum tube research and development efforts going on in the United States was started. Project lists and technical information were published regularly. These accumulated contributions during the war years were so well received that desires were expressed by various persons and organizations to have the Vacuum Tube Development Committee continue after the war. Fears, however, were expressed that cuts in NDRC funds might cause the VTDC's premature demise.

As a result of these fears, proposals were made in 1944 for the Committee to become sponsored jointly by the Army and Navy to insure that, when the war in Europe ended, enough financial support would be available to guarantee that the work of the Committee could continue. This proposal was taken under advisement and at a meeting held at the Pentagon, on January 5, 1945, it was agreed that the Signal Corps of the United States Army would integrate the Vacuum Tube Development Committee as part of its Joint Communications Board, with fifty per cent of its funding supplied by the Navy.

The Joint Communications Board, on March 1,

[2]The full text of the specific recommendations may be obtained from the authors.

45, issued a new directive for the Committee ich differed from the old NDRC directive by the dition of an eighth requirement, "It shall have gnizance over and shall direct engineering, chnical, secretarial and other contractor serv- es provided by the War and Navy Departments in lfillment of the VTDC function."

Fifteen months later, when the Charter was itten for the Joint Research and Development ard (JRDB) on June 6, 1946 and amended on ly 3, 1946, it was noted that specialty panels ould be formed to assist the Board in its activi- es and functions. On August 15, 1946, the JRDB tablished the Committee on Electronics and in- uded in its directive the statement that upon ap- oval of the Joint Chiefs of Staff, the VTDC would transferred to the Committee on Electronics. October 24, 1946, this transfer was accom- ished and the Panel on Electron Tubes (PET) as formed.

The objective of the Panel on Electron Tubes as established by the JRDB as the ". . . achieve- ent of a well-balanced program of research and velopment of electron tubes with a view to the ng-term requirements of the War and Navy De- rtments. The accomplishment of this objective cessitates the continuing study, evaluation, im- ovement and allocation of electron tubes re- arch and development plans, programs, and oblems in the national defense effort and in re- tion to the available and potential store of scien- ic information, personnel and facilities. . . ."

In order to pursue the stated objective, the nel on Electron Tubes was directed to obtain formation about vacuum tube research and de- lopment programs within and outside of the ited States. They were further required to alyze the information they obtained in order to d duplication of effort, to focus ". . . constant nphasis upon the major problems. . . ," to . . determine . . . serious gaps which exist. . .," d to estimate the future requirements for fa- lities, equipment, and personnel.

The PET pursued this program under the JRDB til December 21, 1948, at which time the new search and Development Board, Committee on ectronics, issued a revised directive to the nel on Electron Tubes. The new directive, al- ough not significantly different from the preced- one, did reorient some of its effort. Now, at ast once each year, presentations of an inte- ated program of research and development in cuum tubes for military purposes were re- ired by the new directive. A requirement for aff studies as to the allocation of responsibility r specific programs among the military staff was also included. In general, the broad require- ment of the ". . . achievement of a well-balanced program of research and development of electron tubes. . . " remained.

With the establishment of the Panel on Electron Tubes under the Research and Development Board, the support contract was passed in 1949 to New York University, where it presently resides.

1950—1959

Concomitant with these changes within the or- ganization of the Panel on Electron Tubes, the De- fense Department, through the Research and De- velopment Board, noted that difficulties with elec- tronic equipments were not governed solely by a focus upon electron tubes and their associated cir- cuitry. At this time, as well, various postwar in- vestigations and studies were reaching the conclu- sion that excessive percentages of electronic gear were being received by their military users in un- usable fashion and that those equipments which did work were not working consistently. Considering this increasing evidence, the Research and De- velopment Board, on December 7, 1950, formed the Ad Hoc Group on Reliability of Electronic Equip- ment through its Committee on Electronics.

The Ad Hoc Group was directed by the Com- mittee on Electronics to ". . . 1. Determine the major goals and problems of field maintenance of electronic equipment and direct constant emphasis on greater reliability and equipment designs which would reduce these problems. 2. Summarize the causes of failure in existing equipments, systems, and current procedures. 3. Appraise the industrial potential to meet military requirements in this field. 4. Evaluate the concepts of component design and systems to reduce field maintenance problems. 5. Study existing and proposed research and de- velopment programs in the light of such concepts and make recommendations to the Committee thereof. 6. Recommend areas where additional or new research and development effort is needed."

To fulfill this assignment, Ad Hoc Group mem- bers were appointed from the three military de- partments, the Joint Chiefs of Staff, the Munitions Board, and various civilian organizations and pro- fessions. The Panel on Electron Tubes was in- cluded as an advisor.

During this period that the Ad Hoc Group was active, 1950-1952, the Army and Navy undertook increasing numbers of studies in order to add to their knowledge of electronic equipment failures. For example, the Navy contracted for the Vitro Corporation to investigate component failures; Aeronautical Radio, Incorporated, to investigate

electronic tube failures; and the Bell Telephone Laboratories to study component part failures. The Army, through the Signal Corps, entered into a long-term Tube Analysis Program with Cornell University. The Air Force requested the RAND Corporation to investigate the general electronic reliability problem. These studies were conducted in addition to the military's continuing in-service efforts.

Twenty months after the formation of the Ad Hoc Group, the Chairman of the Research and Development Board, in a letter to the Secretary of Defense dated August 14, 1951, noted that the Research and Development Board had established the Ad Hoc Group on Reliability of Electronic Equipment because of large numbers of reports of unsatisfactory performance of electronic equipment in the field. He defined the objective of the Group as examining the reliability program in the broad sense and stated that he expected that they would recommend measures which should result in reliable performances with a minimum of maintenance. He further noted that the reliability problem really extended far beyond the scope of the Research and Development Board, and, in fact, he felt that it required a combined effort with the Munitions Board, the Joint Chiefs of Staff, and the operating arms of the three services. Accordingly he recommended to the Secretary of Defense that the Department of Defense recognize the broad scope of the reliability problem, that it endorse the Ad Hoc Group's work, and that it enjoin all defense agencies to increase their emphasis on reliability factors. General George C. Marshall, then Secretary of Defense, issued on September 12, 1951, in response to this request, Department of Defense Directive 150.21-1, "Reliability of Electronic Equipment."

He initially reviewed the work of the Ad Hoc Group and stated that ". . . reliability must be a prime objective in all phases of the procurement and use of . . . equipment." He went on to direct that ". . . increased emphasis on reliability of military electronic equipment by all agencies of the Department of Defense is required."

Six months later, on February 18, 1952, the Ad Hoc Group on Reliability of Electronic Equipment issued their final progress report in two volumes, making seventeen major recommendations for action by various agencies. These recommendations were that: 1) failure data reports be compiled on the basis of field use and be summarized, evaluated, and placed in the hands of designers; 2) tube, component, and especially system reliability programs be continued by appropriate groups; 3) reliability requirements be added to military characteristics prepared by the Joint Communication Electronics Committee; 4) a study be made of maintenance minimization; 5) a study be made of effects of unreliable equipment; 6) reliability concepts be involved in procurement, production, and quality control of electronic equipments; 7) the RDB reliability activity be extended from the initiation of military characteristics through its operational use and a permanent RDB reliability group be established; 8) educational activities be expanded and a reliability information center be maintained by the RDB to provide reliability data; 9) testing of equipment, simulating use in the field, be expanded; 10) analyzing and approving new designs with regard to easy maintenance and reliable performance be established; 11) a reliability section be put into specifications; 12) the training of inspectors be improved; 13) engineering supervision of installation of equipment by the material agencies and their contractors be improved; 14) training of operators be improved, with reference to results of operational abuse of equipment; 15) maintenance problems be investigated to secure better preventive maintenance, training and simpler test equipment; 16) reliability organizations be set up in the military department; and 17) classification as to degree of reliability necessary be adopted and integrated into military characteristics.

Six months following, as one consequence of these recommendations, an Advisory Group on Reliability of Electronic Equipment (AGREE) was formed by the Department of Defense on August 21, 1952.

When the Research and Development Board was abolished in 1953, AGREE was transferred to the Assistant Secretary of Defense (Research and Engineering) and re-established in 1954 as part of the Office of the Assistant Secretary of Defense (Applications Engineering). The final directive for AGREE, dated March 31, 1954, set the purpose of AGREE as assuring that ". . . the best available scientific, engineering, production and operational talent are applied to the achievement of reliability in the field of military electronics. The Advisory Group will monitor, stimulate interest in, and advise on, reliability matters within its field of electronic equipment, design, development, procurement, production, maintenance, installation, operations and training."

At the same time, March 30, 1954, the Panel on Electron Tubes was redesignated the Advisory Group on Electron Tubes (AGET) with essentially the same charter.

A few months later, on June 8, 1954, the Advisory Group on Electron Parts (AGEP) was

ormed, with the objective of assisting ". . . in chieving a sound, coordinated, and integrated re- earch and development program in the field of lectronic parts." The field of interest of AGEP as defined as including ". . . research and de- elopment of electronic parts including capaci- ors; coils, inductors, and transformers; electric nd magnetic properties of materials; electro- echanical devices; frequency control devices; esistors; transmission lines; and techniques for ackaged subassemblies utilizing miniature elec- ronic parts and printed circuits." In order to each this objective, AGEP was to ". . . continu- usly observe research and development activities the field of electronic parts, both within and ithout the Department of Defense. . . ."

During the period following the formation of GREE, AGET, and AGEP, the question of unre- ability in military equipment became a legisla- ve issue. In July, 1954, the House of Represen- atives' Committee on Government Operations had efore it for study a report by its Subcommittee n Military Operations concerning the develop- ent and procurement of AN/ARC-21 airborne adio transceivers.[3] The Subcommittee noted at the stress of the Korean War was a major actor in the need for the AN/ARC-21, but felt at the unreliability of the equipment should have een resolved before the Air Force ordered pro- uction equipment. As they stated, "The Subcom- ittee is of the opinion that it is not enough to de- ign radio equipment which meets complex per- ormance requirements. To be of real value, the quipment must be economical in initial cost, op- rating cost, and maintenance costs and in provid- g the flexibility necessary to meet changing op- rational demands."

In March, 1955, the same Subcommittee re- iewed the merits of TAGAN and VOR-DME air avigation equipment.[4] Once again, the reliability f the equipments was discussed and again the ubcommittee noted that the reliability of a piece f gear should be determined before production of e gear is undertaken.

By late 1955, AGREE felt ". . . that sufficient nowledge was available and sufficient interest aroused that specific steps could be taken toward quantifying reliability requirements and toward developing suitable tests to verify that such re- quirements are met. Consequently, a program of nine tasks in the areas of numerical reliability re- quirements, tests, design procedures, components, procurement, packaging and transportation, stor- age and operation and maintenance was established. A task group of members from the Military De- partments and industry was assigned to each of the tasks early in 1956."

As a result of its investigations, a report of major magnitude was issued on June 4, 1957 by AGREE entitled "Reliability of Military Electronic Equipment." The core findings of this report serve as a technical basis for the current approach to military system reliability and are as follows:

Task Group 1 developed minimum acceptability figures for various electronic equipments. These figures, in "mean time between failures" (MTBF), were derived in liaison with operational commands and represented first steps toward a compilation of such calculations.

Task Group 1 also recommended: "As time and effort for additional study become available, these figures be modified with regard to test en- vironment, state-of-the-art, compromise with other performance features, cost, maintenance load and availability. Rather than make no speci- fications in view of the long delays that these addi- tional studies may require, such modifications be made by considered opinions. Special studies be made to establish reliability requirements for the major air defense data-handling systems such as SAGE, Naval Tactical Data System, and MISSILE- MASTER, and for missile-borne electronics equip- ment."[5]

In an Appendix to their report, they provided mathematical bases for their work as well as a cost model for optimizing reliability.

Task Group 2 established a test procedure for design equipments which they felt ". . . balances economy of time and facility against the rigors of high accuracy and risk of wrong decision." On the basis of their studies, the Group recommended that, in addition to the proposed testing, the reli- ability predictions prepared by the contractors should be carefully reviewed and that a review also be made of the contractor's efforts in com- ponent testing and failure follow-up. The failure- rate test, they believed, should not be the sole

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Committee on Government Operations, Subcommittee n Military Operations, 83rd Congress, 2nd Session, Air Force Procurement of Airborne Radio Trans- eivers," House Rept. No. 2578, Washington, D.C.; 954.

Committee on Government Operations, Subcommittee n Military Operations, 84th Congress, 1st Session, Military Procurement of Air Navigation Equipment, 1 nd 8 March 1955," House of Representatives Hearings, ashington, D.C.; 1955.

[5]Advisory Group on Reliability of Electronic Equipment, Office of the Assistant Secretary of Defense (Research and Engineering), "Reliability of Military Electronic Equipment," Washington, D.C.; June 4, 1957.

basis of decision, because: first, the time and numbers available for test are usually quite limited, thus placing a very broad confidence limit on the results; second, developmental models are rarely representative of future production; and third, the failure pattern of the developmental models is rarely representative of all failures. To obviate these points, the group provided details for a careful review of reliability prediction based upon review of paper design and the contractor's programs for component test-to-failure.

Future analysis, they felt, would be made of all test failures in order that adequate corrective action be initiated. The contractor's total reliability effort, they go on, should ". . . be supervised by an independent evaluation group that is not subject to the interests of prejudices of project personnel on the staffs of the contractors or procurement agency."

Task Group 3 established specific routines for ". . . reliability index (mean life) evaluation of pilot-production equipment . . . reliability index evaluation of production equipment, and . . . longevity . . . evaluation of . . . production equipment." These routines permit, according to Task Group 3 ". . . the establishment of the equipment's capability of meeting a minimum reliability requirement . . . statistically conclusive proof that an acceptable percentage of quantity-produced equipment meets a minimum reliability requirement . . . and . . . conclusive proof that equipment reliability does not degrade below a prescribed minimum level during the desired life of the equipment."

The selected testing method theoretically cannot be affected by the contractor or by prejudiced testing personnel. Techniques are established in which the testing methods are relatively self-checking and immune to errors in data recording.

Task Group 4 considered developmental procedures for any equipment so that the equipment would have the required inherent reliability. The proposed developmental program was divided by the Task Group into two phases: a feasibility study which would include a theoretical reliability prediction and which is terminated by the contractor's report of this prediction, and the design and construction of prototype models.

Task Group 5 reported that the then current military component specifications did not assure achievement of required reliability levels. The Qualification Approval lists, they felt, provided no assurance of any determinable failure rate for component reliability and the present military inspection practices did not police reliability levels or yield data for reliability assurance. Group 5, therefore, established a test procedure for ". . . determining the reliability of component parts and tubes . . . in terms of failure rate . . . or in special parametric terms that apply to equipment reliability." They further recommended that a permanent group be established at Department of Defense level to include representatives of industry and the three services and to be charged with the tasks of developing military component specifications, of testing component parts for design capability, and of developing inspection methods.

Task Group 6, in general, found that the present procurement and contracting practices and regulations were inadequate to insure the high-reliability objectives as noted by Task Group 1. They recommended that comprehensive sets of technical specifications be established in order to produce the degrees of reliability required. They recommended that procurement agencies use the findings of Task Groups 1 through 5 in the form of specifications of equipment in order to assure the procuring of reliable equipments.

Task Group 7 recommended, as a result of their investigations, that the equipment designer and the package designer meet early and often to determine the best packaging possible. Feedback from studies on shock and vibration during handling and transportation should become more effective than at present. The military should enforce all of the requirements for bracing and blocking. They further recommended that specifications be written to cover test procedures for the simulation of transportation and handling environment.

Essentially, Task Group 8 found that failures caused by storage are not significant when compared with failures caused by other things. However, they believe that this conclusion is based upon inadequate records and, therefore, recommended that more careful records be kept in order to determine the cause of failure of equipment.

Task Group 9, in reviewing the methods and procedures for maintaining the reliability of equipment in service, made a careful study of equipment maintainability, performance checking, disposable modular units, test equipment calibration, as well as maintenance publications, manuals and handbooks. Included also in its studies were preventive maintenance and marginal checking, the shortage of technicians and the general education of engineers. A large number of recommendations were made. They felt that all contracts to be awarded by the Department of Defense should include a quantitative maintainability requirement and that the contractor should be required to demonstrate by test that his equipment has, in fact, met this particular maintainability requirement.

verage-skill military technicians should be vailable to perform maintenance during these quipment evaluation tests. Maximum training ime should be limited to approximately a third of ny serviceman's remaining enlistment. Preventive maintenance should be limited to only those omponents and parts which obey a wear-out law f failure. Marginal checking should be used vhenever possible. Calibration centers for test quipment should be established at various locaions to supplement existing facilities; standards gainst which these equipments are calibrated hould be regularly compared with those available t the National Bureau of Standards.

Subsequent to the publication of the report, it vas realized that efforts would have to be made o complete the work started by Task Group 5. ccordingly, in February, 1958, action was taken o begin such an Ad Hoc effort. On July 14, 1958, n agreement was reached between the Director f Production Policy, OASD (Supply and Logistics) nd the Director of Electronics, OASD (Research nd Engineering),[6] for establishing the Ad Hoc tudy Group on Parts Specification Management or Reliability. The basic objective of the Ad Hoc tudy Group was to ". . . analyze the recommenlations established by the AGREE Task Group 5 n order to advise the Assistant Secretaries of Defense (Research and Engineering) and (Supply nd Logistics) regarding efficient implementation nethods and procedures."

Expanding upon this objective, the Group conidered specification preparation, requirements, upport from industry and documentation, along vith questions of Qualified Product Lists and the eed for a management organization for military art specifications at the Department of Defense evel.[7]

During the Sixth National Symposium on Reliaility and Quality Control in Electronics, Janury 11-13, 1960, some of the conclusions reached y the Ad Hoc Study Group were published.[8] Three prototype specifications were presented vhich include some of the new features the Ad Ioc Group proposes. They provide ". . . four or ive reliability levels" as well as ". . . life test ampling plans" for various components. The Group noted that qualification approval procedures should be modified such that approvals will be granted for one of the reliability levels. If the component is improved and does reach a higher reliability level, it then will be raised on the list. Requalifications should be required every twelve months.

Three months prior to the publication of the AGREE report, in March, 1957, the Ad Hoc Committee for Guided Missile Reliability (ACGMR) was formed under the Assistant Secretary of Defense, Research and Engineering, and on November 15, 1957, was transferred to the Office of the Director of Guided Missiles. The ACGMR was to design a ". . . uniform monitoring program and management procedure that can be effectively used for all types of guided missile projects."

In April, 1958, the ACGMR published their report. The management and monitoring program ACGMR devised starts when the contract is awarded and continues through all phases of design, development, production and major product improvement. To insure compliance with reliability specifications and to aid the contractor in knowing whether these goals are being reached, eight test points are established, at which time a fully documented report from the contractor, concerning either the predicted or the verified reliability, is required.

The first monitoring point, Detail Design Study, starts "with the contract award and ends with a design report that includes studies of system and subsystem reliability that encompass the entire weapon-system design and includes an assessment of reliability, using prediction techniques wherever feasible."

The second monitoring point, Preprototype, occurs when ". . . the initial system design is nearly complete and many component parts and assemblies have undergone some developmental testing. This point may be identified by some such phrase as '95 per cent of engineering released,' 'design engineering inspection' or 'the time at which initial design is essentially complete.' "

The third monitoring point, Prototype, occurs when the first complete sets of hardware or subsystem hardware are available and ". . . can be assembled into the general physical configuration which they will have when used by the Military Services. Laboratory testing has been conducted to demonstrate the compatibility of weapon system and subsystems. Special test-vehicle flights to obtain data for design improvement are performed. During this phase, all necessary research and engineering data are obtained and the basic design firmly established."

E.J. Nucci, "Progress report on ad hoc study on parts pecifications management for reliability," 1959 IRE ATIONAL CONVENTION RECORD, pt. 6, pp. 120-129.
R. Soward, "Status Report on Department of Defense d Hoc Study Group on Parts Specifications Management or Reliability," September 18, 1959.
R.E. Moe, "Improved component and tube specificaions," Proc. Sixth Natl. Symp. on Reliability and Quali- Control, Washington, D.C., pp. 1-11; January 11-13, 960.

The fourth reliability monitoring point, Preproduction Demonstration, occurs when "... the production design of the weapon system is essentially complete and the missile or missile system is considered ready for production. A demonstration of the reliability achieved during this stage provides one of the bases for assessing the system's readiness for full-scale production."

The fifth monitoring point, Demonstration of Service Readiness, causes the Contractor to "... show that the weapon system which is usually built under the limited- or pilot-production program has reached the reliability objectives—that the system can be produced in quantity without significant loss in performance or reliability."

The sixth monitoring point, Service Evaluation, is performed by military personnel. As stated, "... the military service uses its own personnel to perform its own weapon-system evaluation tests. If the weapon system is found to be operationally acceptable and is capable of being produced in quantity without significant loss in performance or reliability, approval of production for service use is usually given at this monitoring point."

The seventh monitoring point, Full-Scale Production, insures that "... the level of reliability designed into the system is maintained during production."

The eighth and last reliability monitoring point, Demonstration of Major Product Improvement, occurs when "... the reliability and over-all value of major product improvements are demonstrated and may be approved for incorporation into the weapons system."

These eight reliability monitoring points are based upon several conclusions of the Committee. First, the Committee believes that "... reliability is a parameter that can be predicted, assessed, measured and controlled during the design, development, production and major product improvement phases of guided-missile weapon systems." Second, they believe that "... it is technically feasible and sound to specify and monitor reliability in guided-missile weapon systems during their growth cycle."

This ACGMR report had immediate consequences, since it provided the military services with a document detailing the management procedure by which they could implement these technical recommendations of such committees as AGREE, AGEP and AGET to assure required reliability in complex military systems.

Epilog

If a view of the future lies in the past, then certain generalities appear within the realm of tomorrow. Military systems will depend increasingly upon electronics, while electronic system failures, running second only to human factor failures, may contribute disproportionately to the uncertainty of future military efforts. It is more than likely that the Department of Defense will further its concern with the scientific research and development necessary to raise military system reliability. And, as a corollary, such attention will continue to focus upon electronics, so long as it appears to account for an excessive portion of total system failure.

The Military question will no longer be "Will the system do the job?" Now the challenging question will be "How confident can we be that the system will do the job when and for as long as it is needed?"

Breaking Even on Failure Rate Reduction

J. B. HEYNE†, SENIOR MEMBER, IRE

Repeated studies of the time incidence of component failures in complex machines indicate an apparent exponential relationship between the probability of failure-free operation, P_o, and the time at which failures are expected to occur, t. When these are plotted against each other, it is to be expected that the slope of the curve at t = 0 is $-\lambda$:

$$P_o = e^{-\lambda t} \tag{1}$$

where λ is the mean failure rate.

This relationship is predicated on several assumptions. The first of these is that all failures are random and occur in accordance with a Poisson distribution. All other contributory causes of failure are presumed to have been eliminated by design review or by the time-honored engineering practice of "debugging." Because the empirical data which support this exponential relationship were derived under certain conditions, it is necessary that these same conditions apply when the exponential relationship is used for the prediction of machine expectation. These conditions are 1) that the machine be subjected to continuous and/or repeated operation, 2) that the machine be subjected to continuous and/or repeated service, and 3) that the environmental stress be similar to that under which the basic data were gathered.

In determining the probability of a machine being able to fulfill its operational requirement, it is exceedingly important to point out that the occurrence of a single component failure is rarely enough cause for complete machine breakdown. Moreover, depending on the particular structure of the machine and on the location of failed components, more than one component failure can occur without causing machine breakdown. This characteristic lends itself to mathematical development if the following quantities are defined:

P_o = the probability of failure-free operation of a complete machine

P_i = the probability of failure-free operation of the ith function

Q_i = the probability of failed operation of the ith function (= $1 - P_i$)

R_i = the effectiveness of the machine under the conditions that the ith function has failed and all other functions are operable

R_o = the effectiveness of the machine under failure-free conditions; for most purposes this quantity can be considered to be unity.

It follows then that the total capability of the machine, viewed in the light of operating in the face of partial failure is

$$P_c = P_o R_o + \sum_i P_o \frac{Q_i}{P_i} R_i \tag{2}$$

where i can be taken to describe a particular machine function. The P_i factors may be determined from the components merely by adding the respective mean failure rates of those components which lie in each functional path and applying the sum in the exponential formula. Rigorous mathematical representation requires that the probability of simultaneous failure be included. This probability tends to be so low as to become insignificant within the accuracy of presently measurable mean failure rates.

The total capability of an operator-controlled machine can be measured as a function of the information presented to the operator. If it is assumed that complete presentation of information defines maximum capability, it becomes possible to quantify the relative loss of capability which results when various bits of information cannot be presented. A method for establishing system capability on the basis of audio-visual displays and controls to the operator can be inferred here. The execution of such a calculation requires detailed analyses such as can best be performed by those close to the design of the machine.

The expectation, E, of satisfactory machine performance when required can then be determined as the product of three probabilities: the probability that a machine is neither awaiting nor undergoing service; the probability that a machine which is neither awaiting nor undergoing service is truly ready; and the probability that a machine which is truly ready will be able to fulfill its

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operational requirement.

Another measure of expectation is that it is equal to the ratio of the number of machines required to fulfill an operational requirement to the number of machines which must be initially procured in anticipation of the operational requirement:

$$E = P_{OL}P_aP_c{}^1 \tag{3}$$

where

$$P_{OL} = \frac{T_t - T}{T_t}, \tag{4}$$

P_a is the assessibility

$$P_c = P_o + \Delta = e^{-\lambda t} + \sum P_o \frac{Q_i}{P_i} R_i \,. \tag{5}$$

Let

$$\Delta = \sum P_o \frac{Q_i}{P_i} R_i \,. \tag{6}$$

Let $N_n = EN$ where N_n is the number of machines necessary to meet an operational requirement and N is the number of machines procured in anticipation of the operational requirement. Further, let $N_n = E'N'$ where E' and N' have the same definition as before but that their values have been modified by changing one of the factors which contribute to down time, in this case λ, the mean rate at which replacements become necessary:

$$N_n = EN = E'N' \tag{7}$$

$$\frac{N}{N'} = \frac{E'}{E} = \frac{P'_{OL}P'_aP'_c}{P_{OL}P_aP_c} = \frac{(T_t - T')P'_a(P'_o + \Delta')}{(T_t - T)P_a(P_o + \Delta)}$$

$$= \frac{(T_t - T')(P'_o + \Delta')}{(T_t - T)(P_o + \Delta)} \,. \tag{8}$$

It is assumed that $P'_a = P_a$ when only the mean replacement failure rate is changed:

$$T' = T + (\lambda' - \lambda)\frac{\delta T}{\delta \lambda} \tag{9}$$

so that

$$\frac{(T_t - T')}{(T_t - T)} = 1 - \left(\frac{\lambda - \lambda'}{T' - T}\right)\frac{\delta T}{\delta \lambda} \tag{10}$$

where $\lambda' - \lambda$ is the change effected in the mean replacement failure rate:

$$\frac{N}{N'} = \left[\frac{e^{-\lambda' t} + \Delta'}{e^{-\lambda t} + \Delta}\right]\left[1 - (\lambda' - \lambda)\frac{dT}{d\lambda}\left(\frac{1}{T_t - T}\right)\right]. \tag{11}$$

The change in the necessary number of initial machines which owes itself to the change from λ to λ' may be written as

$$\frac{N}{N'} - 1 = \left[\frac{e^{-\lambda' t} + \Delta'}{e^{-\lambda t} + \Delta}\right]$$

$$\left[1 - (\lambda' - \lambda)\frac{dT}{d\lambda}\left(\frac{1}{T_t - T}\right)\right] - 1 \tag{12}$$

where T and $\frac{\delta T}{\delta \lambda_b}$ are developed as follows.

Machines are not on-line during the period they are awaiting or undergoing service. Service may be scheduled, occurring at regular time intervals, or it may be unscheduled necessitated by the random occurrence of machine failure:

$$T = T_p + T_u \tag{13}$$

where

T = mean total down time expected during a given time period T_t

T_p = mean total time in scheduled service expected during T_t

T_u = mean total time in unscheduled service expected during T_t.

The mean total time in scheduled service during T_t is the sum of the following five parts:

1) The mean number of scheduled servicings, N_p, multiplied by the mean total time necessary to unbutton the machine, hook up support equipment used in scheduled service, unhook support equipment and rebutton the machine. Part 1), of scheduled service may be written as:

$$N_p t_a = \frac{\psi}{M} t_a \tag{14}$$

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[1]Where P_a is defined as the ratio of the number of components energized in self-test to the total number of components, this equation becomes

$$E = P_{OL}P_aP_c + P_{OL}(1 - P_a)\sum_j P_o \frac{Q_j}{P_j} \cdot R_j$$

when the jth machine function contains components in its path which are not energized in self-test. In this case, the ith machine functions are presumed to contain only those components which are energized in self-test.

where

ψ = the mean total number of operational hours expected on the system during T_t

M = the mean total number of operational hours allowed between scheduled servicings

t_a = the mean total time required to unbutton, hook up, unhook and rebutton the machine at each servicing.

2) The mean number of scheduled servicings multiplied by the mean time required to perform service on the machine, provided that no adjustments or replacements are necessary. Part 2) of scheduled service may be written as:

$$N_p t_b = \frac{\psi}{M} t_b \tag{15}$$

where

t_b = the mean total time required to perform service on a machine which has been rendered accessible, provided that no adjustments or replacements are necessary.

3) The mean number of replacements found necessary to the machine at the beginning of scheduled service, N_b, multiplied by the mean total time necessary to make each replacement. This is a function of the probability of detecting a failure during scheduled service called "thoroughness," and of the probability that a given replacement will not require further adjustment called "interchangeability," and of the probability of having detected a failure other than during scheduled service called "assessibility." Thus,

$$N_b(1 - P_a)\ P_e T_b$$

where

N_b = the mean total number of replacements necessary at the beginning of scheduled service

P_a = the probability of failure detection at times other than scheduled service called assessibility

P_e = the probability of failure detection during scheduled service of time duration t_b, this may be called thoroughness

T_b = the mean total time required for a replacement to be made.

This may also be written as

$$N_b(1 - P_a)\ P_b P_e T_b + N_b(1 - P_a)(1 - P_b) P_e(T_a + T_b)$$

where

P_b = the probability that a given replacement does not require further adjustment called interchangeability

T_a = the mean total time required for an adjustment to be made

λ_b = defined as the mean rate at which replacements are necessary to the machine.

Part 3) of scheduled service may be written as:

$$M\lambda_b(1 - P_a)\ P_b P_e T_b$$
$$+ M\lambda_b(1 - P_a)(1 - P_b)\ P_e(T_a + T_b)\,. \tag{16}$$

4) The mean number of adjustments found necessary at the beginning of scheduled service, N_a, multiplied by the mean total time required for an adjustment to be made. Previous scheduled service is assumed to have insured that all necessary adjustments were made. P_d is the probability that a given adjustment will obtain during M operational hours called "adjustability." Part 4) of scheduled service may be written as:

$$P_d P_e N_a T_a \tag{17}$$

where

λ_a = the mean rate at which adjustments are necessary to the machine. Where $\lambda_a \leq 1/M$, $P_d = 1$. P_d is less than unity whenever $\lambda_a \geq 1/M$.

5) The mean number of replacements necessitated by the fact that the machine is being operated during the scheduled service, multiplied by the mean total time required for each replacement. The environmental stress on the machine during scheduled service is such that it can be assumed that no failure resulting from loss of adjustment will occur once the adjustment has been made, and that any failure which occurs will be of the type which requires replacement. Further, it shall be assumed that the nature of scheduled service affords such minimum environmental stress on the machine and is of such relatively short time duration, t_a, that no failures will occur. Part 5) of scheduled service may be written as:

$$sN_g P_b P_e T_b + sN_g(1 - P_b)\ P_e(T_a + T_b) \tag{18}$$

where

s = the mean ratio of environmental stress on the machine during service to the environmental stress on the machine during operation.

N_g = the mean number of replacements necessitated during that part of scheduled service where environmental conditions might be expected to cause failures.

The mean total time in unscheduled service during T_t is made up of the following three parts:

1) The mean total number of adjustments found to be necessary during the time period between scheduled services, multiplied by a time which is the sum of: a) the mean total time required to make an adjustment; and b) the mean time, t_c, which is required to detect a failure which has occurred between regularly scheduled services; plus c) t_a, basic setup and set-down times. Part 1) of unscheduled service may be written as:

$$N_a(1 - P_d)(T_a + t_a + t_c). \tag{19}$$

2) The mean total number of replacements found to be necessary during machine operation between scheduled services, multiplied by the sum of mean total times to setup, detect failures, adjust and replace. This mean number of replacements is a function of the assessibility, P_a, and the interchangeability, P_b. Part 2) of unscheduled service may be written as:

$$N_bP_aP_b(T_b + t_a + t_c) + N_bP_a(1 - P_b)(T_a + T_b + t_a + t_c). \tag{20}$$

3) The mean total number of adjustments and replacements which were actually necessary at the time of scheduled service but which were missed because the thoroughness, P_e, is less than unity. Part 3) of unscheduled service may be written as:

$$(1 - P_e)[T_p - \frac{\psi}{M}(t_a + t_b)]. \tag{21}$$

By combining the preceding equations an expression for the total down time, T, is derived

$$T = \frac{\psi}{M}(t_a + t_b) + \frac{a_1T_aT_b + a_2T_b^2 + a_3T_aT_b^2 + a_4T_a^2T_b + a_5T_b^3 + a_6T_b + a_7T_a}{T_b(1 - \lambda_b sP_eT_b - \lambda_b s(1 - P_b)P_eT_a)}. \tag{22}$$

Eq. (22) enables the evaluation of the expected effect on the mean total down time which variation of any of the contributing down-time parameters might cause. If λ is the down-time parameter whose variation is under study,

$$\frac{\delta T}{\delta\lambda} = \frac{b_1T_aT_b^2 + b_2T_aT_b^3 + b_3T_a^2T_b^2 + b_4T_b^3}{\omega^2} + \frac{b_5T_b^4 + b_6T_b^5 + b_7T_aT_b^4 + b_8T_a^2T_b^3}{\omega^2} + \frac{b_9T_a^3T_b^2 + b_{10}T_b^2 + b_{11}T_aT_b + b_{12}T_a^2T_b}{\omega^2} \tag{23}$$

where, for convenience,

$$\omega = T_b[1 - \lambda_b sP_eT_b - \lambda_b s(1 - P_b)P_eT_a] = T_b - \beta T_b^2 - \phi T_aT_b \tag{24}$$

$$\beta = \lambda_b sP_e. \tag{25}$$

Algebraic expressions for these constants have been derived and are available to those requesting them. They have been omitted here for convenience.

The insertion of nominal or measured quantities for the λ's, P's, T's, M's and s establishes values for T and for $\frac{\delta T}{\delta\lambda}$. Once these are established, it is possible to measure the value of any planned or expected change in λ in terms of a reduction in the number of initially procured machines necessary to the fulfillment of an operational requirement. Where the cost of reducing the mean replacement failure rate, λ, is less than the expected savings resulting from the procurement of fewer initial systems, this means of improving expectation should be explored. This is true for all of the down-time parameters and provides a basis for deciding which down-time parameter will yield the greatest improvement in over-all expectation for a given investment of resources. Calculations of expected savings in terms of machines should include the cost of supporting such additional machines as well as the cost of their initial procurement.

Optimal Diagnostic Procedures*

B. B. WINTER†

Summary.—In recent papers,[1,2] optimal diagnostic procedures are presented for some special cases. In this paper, we present an optimal diagnostic procedure under a different restriction, i.e., we consider equipment in which elements can only be tested one at a time, or all at once. Optimality is in the sense of minimum expected cost.

I. MODEL AND TERMINOLOGY

Consider an equipment which consists of N, $N > 1$, elements (as defined by Brulé, et al.[1]). The elements fail independently. The failure of one element does not cause the equipment to cease functioning (though it is now functioning "erroneously") and, thus, can be followed by failures of other elements at subsequent times. An over-all test can be applied to the equipment, such that it passes if and only if all elements are good. At some time, following the failure of at least one element, the equipment is subjected to diagnosis in the following manner.

With the equipment known to be bad (i.e., one or more elements are bad), we test the elements one at a time. Whenever we encounter a bad element, we replace (or repair) it and then apply the over-all test to determine whether we should continue with the testing of individual elements. If, subsequent to the replacement of some element k, the equipment fails on the over-all test and elements $k + 1, k + 2, \ldots, N - 1$, are all found good, then the element N is known to be bad "by elimination" and need not be actually tested. Furthermore, if the element N is thus found to be bad, the over-all test need not be performed after replacement of that element. The last assertion arises from the implicit assumption that no element can fail during diagnosis.

Since the equipment continues functioning after the failure of one or more elements, we can only detect equipment failure by periodic examinations of the equipment (such examinations can, of course, be performed by application of the over-all test). When the equipment is found bad by such an examination, we then engage in the diagnostic procedure described above.

Let $p_i(t)$ be the a priori probability that element i is bad, if the equipment passed examination at time t_o and is about to be examined at time t; then

$$p_i(t) = \int_{t_o}^{t} F_i'(t)dt/[1-F_i(t_o)]$$

$$= [F_i(t)-F_i(t_o)]/[1-F_i(t_o)],$$

where F_i is the failure distribution function for element i. If element i is a replacement element, present in the equipment only since some time y_i, then

$$p_i(t) = [F_i(t-y_i)-F_i(t_o-y_i)]/[1-F_i(t_o-y_i)].$$

Note that $p_i(t)$ is a conditional probability, conditional on the event of passing at time t_o, but is a priori with respect to the state of affairs at t since it refers to the probability prior to any knowledge as to whether the equipment is bad at t. In the sequel, we write p_i for $p_i(t)$ and q_i for $1-p_i(t)$.

With each test or repair we associate a fixed "cost," e.g., length of time to perform the test or repair in question. The quantity which is taken to be the cost of an operation must have the properties:

1) the cost associated with an operation takes on the value zero if the operation is not performed, and takes on some fixed value if the operation is performed.
2) the costs are linear; the cost associated with a complete testing and repairing sequence is the sum of the costs incurred in connection with the individual operations.

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1 J.D. Brulé, R.A. Johnson, and E.J. Kletsky, "Diagnosis of equipment failures," IRE TRANS. ON RELIABILITY AND QUALITY CONTROL, vol. RQC-9, pp. 23-34; April, 1960.

2 R.A. Johnson, "An information theory approach to diagnosis," Proc. Sixth Natl. Symp. on Reliability and Quality Control, Washington, D.C.; January 11-13, 1960.

The following notation is used, with $i = 1, 2, \ldots, N$:

$f(S_k)$	expected cost of the testing sequence S_k;
T	cost of over-all test;
τ_i	cost of testing ith element;
ρ_i	cost of repairing (or replacing) ith element;
p_i	a priori probability that element i is bad;
$q_i = 1 - p_i$	a priori probability that element i is good;
$Q_i = \prod_{j=i}^{N} q_j$	a priori probability that elements i, i+1, ... N are all good.

To avoid triviality, we require $0 < p_i < 1$, all i. As a convenience, we write

$$p_o = \tau_o = \rho_o = 0 \quad \text{and} \quad q_o = 1$$

for a fictitious zeroth element.

The numbering of the elements is fundamentally arbitrary. As a convenience, let us require that the elements be numbered in such a manner that

$$\left.\begin{array}{ll} & 1 \leq i < j \leq N \\ \text{implies} & \\ & \dfrac{\tau_i}{p_i} q_i < \dfrac{\tau_j}{p_j} q_j \end{array}\right\} . \qquad (1)$$

II. EXPECTED COST

Let S_N denote the testing sequence in which the elements are tested in the order in which they are numbered, i.e., element No. 1 is tested first, etc. (This sequence is in fact implied in the model description in Section I.)

Let us introduce the set of stochastic variables $\{x_i \mid i=1, 2, \ldots, N\}$ to represent the cost actually incurred in connection with each of the elements, i.e., testing it and, if necessary, repairing or replacing it. Then, for $1 \leq i < N$,

$$x_i = \begin{cases} 0 & \text{if the testing sequence terminated before element i;} \\ \tau_i & \text{if the testing sequence did not terminate before element i, and that element is good;} \\ \tau_i + \rho_i + T & \text{if the testing sequence did not terminate before element i, and that element is bad;} \end{cases}$$

and

$$x_N = \begin{cases} 0 & \text{if the testing sequence terminated before element N;} \\ \rho_N & \text{if the testing sequence did not terminate before element N.} \end{cases}$$

Now, we seek to establish the probability with which each x_i takes on each of its possible values. Since the elements fail independently, the condition that the equipment was good at t_o does not affect the multiplication rule. However, the condition that the equipment is bad at t eliminates from the event space the possibility of all elements being good at t.

Let A denote the event "$x_i = \tau_i$;" let B denote the event "$x_i \neq 0$;" let E denote the event "the equipment is bad." Note that A implies B which, in turn, implies E. Therefore

$$\Pr\{A\} = \Pr\{A \text{ and } B\} = \Pr\{A \text{ and } B \text{ and } E\}.$$
$$\Pr\{B\} = \Pr\{B \text{ and } E\}$$

Also note that, for any events K, L and M,

$$\Pr\{K \text{ and } L \mid M\} = \Pr\{L \mid M\}\Pr\{K \mid L \text{ and } M\}$$

and

$$\Pr\{K \mid L \text{ and } M\} = \Pr\{K \text{ and } L \text{ and } M\}/\Pr\{L \text{ and } M\}.$$

Therefore

$$\Pr\{A \mid E\} = \Pr\{A \text{ and } B \mid E\} = \Pr\{B \mid E\}\Pr\{A \mid B \text{ and } E\}$$
$$= \Pr\{B \mid E\}\frac{\Pr\{A \text{ and } B \text{ and } E\}}{\Pr\{B \text{ and } E\}}$$
$$= \Pr\{B \mid E\}\frac{\Pr\{A \text{ and } B\}}{\Pr\{B\}}$$
$$= \Pr\{B \mid E\}\Pr\{A \mid B\},$$

i.e., for $1 \leq i < N$,

$$\Pr\{x_i = \tau_i \mid E\} = \frac{1-Q_i}{1-Q_1} \cdot \frac{q_i(1-Q_{i+1})}{1-Q_i} = \frac{q_i - Q_i}{1-Q_1}.$$

Similarly,

$$\Pr x_i = \{\tau_i + \rho_i + T \mid E\} = \frac{1-Q_i}{1-Q_1} \cdot \frac{p_i}{1-Q_i} = \frac{p_i}{1-Q_1},$$

and

$$\Pr\{x_i=0\,|\,E\} = 1-\Pr\{x_i \neq 0\,|\,E\} = 1 - \frac{1-Q_i}{1-Q_1} = \frac{Q_i-Q_1}{1-Q_1}$$

$$\Pr\{x_N=\rho_N\,|\,E\} = \frac{1-q_N}{1-Q_1} \text{ and } \Pr\{x_N=0\,|\,E\} = \frac{q_N-Q_1}{1-Q_1}.$$

To summarize, we have, with the equipment known to be bad,

$$x_i = \begin{cases} 0 & \text{with probability } \dfrac{Q_i-Q_1}{1-Q_1} \\ \tau_i & \text{with probability } \dfrac{q_i-Q_i}{1-Q_1} \\ \tau_i+\rho_i+T & \text{with probability } \dfrac{p_i}{1-Q_1} \end{cases} \quad i=1,2,\ldots,N-1$$

and

$$x_N = \begin{cases} 0 & \text{with probability } \dfrac{q_N-Q_1}{1-Q_1} \\ \rho_N & \text{with probability } \dfrac{1-q_N}{1-Q_1} \end{cases}$$

Now we have, for the expected cost,

$$f(S_N) = E(\sum_{i=1}^{N} x_i) = \sum_{i=1}^{N} E(x_i),$$

$$(1-Q_1)f(S_N) = \sum_{i=1}^{N-1} [\tau_i(q_i-Q_i)+(\tau_i+\rho_i+T)p_i]+\rho_N p_N$$

$$= \sum_{i=1}^{N} \rho_i p_i + T\sum_{i=1}^{N-1} p_i + \sum_{i=1}^{N-1} \tau_i(1-Q_i)$$

$$= \sum_{i=1}^{N} \rho_i p_i + T\sum_{i=1}^{N-1} p_i + \sum_{i=1}^{N} \tau_i(1-Q_i) - \tau_N(1-q_N)$$

$$= \sum_{i=1}^{N} \rho_i p_i + T\sum_{i=1}^{N} p_i + \sum_{i=1}^{N} \tau_i(1-Q_i) - p_N(\tau_N+T). \tag{2}$$

III. OPTIMIZATION

Any testing sequence can be derived from the sequence S_N by successive permutations of adjoining elements. To say that elements i and $i+1$ are permuted with respect to S_N is to say that the elements are to be tested in the order $1,2,\ldots,i+1, i,\ldots,N$.

Consider the sequence S_N', obtained from S_N by permuting the adjoining elements r and s, $s \equiv r+1 < N$. We have

$$(1-Q_1)f(S_N') = \sum_{i=1}^{N} \rho_i p_i + T\sum_{i=1}^{N} p_i - p_N(\tau_N+T)$$

$$+ \sum_{i=1}^{r-1} \tau_i(1-Q_i) + \tau_s(1-Q_r)$$

$$+ \tau_r(1-q_r Q_{s+1}) + \sum_{i=s+1}^{N} \tau_i(1-Q_i)$$

$$= \sum_{i=1}^{N} \rho_i p_i + T\sum_{i=1}^{N} p_i - p_N(\tau_N+T) + \sum_{i=1}^{N} \tau_i(1-Q_i)$$

$$+ \tau_s(1-q_r q_s Q_{s+1}) + \tau_r(1-q_r Q_{s+1})$$

$$- \tau_r(1-q_r q_s Q_{s+1}) - \tau_s(1-q_s Q_{s+1})$$

$$= (1-Q_1)f(S_N) + Q_{s+1}[\tau_s q_s(1-q_r) - \tau_r q_r(1-q_s)]$$

$$= (1-Q_1)f(S_N) + p_r p_s Q_{s+1}\left(\frac{\tau_s}{p_s}q_s - \frac{\tau_r}{p_r}q_r\right). \tag{3}$$

Now, since $r < s$ and since the elements are numbered according to (1),

$$\frac{\tau_s}{p_s}q_s - \frac{\tau_r}{p_r}q_r \geq 0;$$

also,

$$p_r p_s Q_{s+1} > 0 \quad \text{and} \quad 1-Q_1 > 0;$$

therefore (3) implies

$$f(S_N') \geq f(S_N).$$

Thus 1) no sequence in which two adjoining elements i-1 and i, $i < N$ fail to satisfy condition (1) can have a lower expected cost than the sequence S_N; and 2) any sequence in which two adjoining elements i-1 and i, $i < N$ do satisfy condition (1) has the same expected cost as the sequence S_N.

Therefore, we have shown that the sequence S_N is optimal among all sequences in which element N is tested last.

Let us examine the expected cost of testing the elements in the order 1,2,...,k-1,k+1,...,N,k, a testing sequence which will be referred to as S_k. Note that S_k differs from S_N only in that element k is tested last. We have, by analogy with (2),

$$
\begin{aligned}
(1-Q_1)f(S_k) &= \sum_{i=1}^{N} \rho_i p_i + \sum_{i=1}^{k-1} \tau_i (1-Q_i) \\
&\quad + \sum_{i=k+1}^{N} \tau_i(1-q_k Q_i) + T\left(\sum_{i=1}^{N} p_i - p_k\right) \\
&= \sum_{i=1}^{N} \rho_i p_i + \sum_{i=1}^{N} \tau_i(1-Q_i) + \sum_{i=1}^{N} p_i \\
&\quad + \sum_{i=k+1}^{N} \tau_i(1-q_k Q_i) - \sum_{i=k}^{N} \tau_i(1-Q_i) - Tp_k \\
&= \sum_{i=1}^{N} \rho_i p_i + \sum_{i=1}^{N} \tau_i(1-Q_i) + T\sum_{i=1}^{N} p_i \\
&\quad + \sum_{i=k}^{N} \tau_i Q_i(1-q_k) - \tau_k(1-q_k Q_k) - Tp_k \\
&= \sum_{i=1}^{N} \rho_i p_i + T\sum_{i=1}^{N} p_i + \sum_{i=1}^{N} \tau_i(1-Q) \\
&\quad + p_k\left[\sum_{i=1}^{N} \tau_i Q_i - T\right] - \tau_k(1-q_k Q_k). \qquad (4)
\end{aligned}
$$

Since the first three sums in the last number of (4) are independent of k, $f(S_k)$ is a minimum when

$$g(S_k) = \min_{1\leq j\leq N}\{g(S_j)\},$$

where

$$g(S_j) = p_j\left[\sum_{i=j}^{N} \tau_i Q_i - T\right] - \tau_j(1-q_j Q_j). \qquad (5)$$

These findings may be summarized in the following algorithm: Number all the elements so that condition (1) is satisfied. Test the elements in the sequence 1,2,...,k-1, k+1,...,N,k where k is determined by condition (5). The expected cost of this testing sequence is a minimum, and is given by (4).

IV. SIMPLIFICATIONS AND EXTENSIONS

A. Inclusion of Superfluous Tests

Assume that we do not rely on testing the last element "by elimination," i.e., we test it even in instances in which the last element's failure can be inferred from outcomes of previous tests. Furthermore, let us perform the over-all test after all elements have either been found good or have been repaired (even though this test is now superfluous). As in the original model, we let x_i be the cost actually incurred in connection with element i; we then have

$$
x_i = \begin{cases} 0 & \text{with probability } \dfrac{Q_i - Q_1}{1-Q_1} \\[2ex] \tau_i & \text{with probability } \dfrac{q_i - Q_i}{1-Q_1} \\[2ex] \tau_i + \rho_i + T & \text{with probability } \dfrac{p_i}{1-Q_1} \end{cases}, \quad i=1,2,\ldots, N
$$

and

$$(1-Q_1)f(S_N) = \sum_{i=1}^{N} \tau_i(1-Q_i) + \left(T + \sum_{i=1}^{N} \rho_i\right) \sum_{i=1}^{N} p_i. \qquad (6)$$

Define S'_N as before, but with $s \equiv r+1 \leq N$ (in the original model, strict inequality held); we find that

$$f(S'_N) \geq f(S_N),$$

thus showing that the sequence S_N is optimal among all possible testing sequences. This result yields the following algorithm: Number all the elements so that condition (1) is satisfied. Test

ne elements in the sequence 1,2,...,N and in accordance with the above assumptions. The expected cost of this testing sequence is a minimum, nd is given by (6).

. Single Element Failure

Assume that one and only one element is bad when the equipment is subjected to diagnosis. This assumption can be made if and only if 1) the quipment fails as soon as any one element fails; nd 2) no elements can fail after the first element ailure.

It is conceivable that equipment satisfying asumptions 1) and 2) can have the further property hat any equipment malfunction becomes immeditely apparent, thus precluding the necessity of eriodically examining the equipment for possible malfunction. If that is the case, we have

$$p_i = F_i(t).$$

If the equipment does have to be examined eriodically (due to the absence of automatic failre indication), then p_i is as given in Section I.

In this model, we let x be the total cost of loating and repairing the single bad element. The onditional probability that element i is bad, iven that one and only one element is bad, is

$$\frac{p_i \prod_{j\neq i} q_j}{\sum_{k=1}^{N} p_k \prod_{j\neq k} q_j} = \frac{\frac{p_i}{q_i}\prod_{j=1}^{N} q_j}{\prod_{j=1}^{N} q_j \sum_{k=1}^{N}\frac{p_k}{q_k}} = \frac{\frac{p_i}{q_i}}{\sum_{k=1}^{N}\frac{p_k}{q_k}}$$

Then

$$x = \rho_i + \sum_{j=1}^{i} \tau_j \quad \text{w.p.} \quad \frac{\frac{p_i}{q_i}}{\sum_{k=1}^{N}\frac{p_k}{q_k}} \quad (i=1,2,\ldots,N).$$

As before, let S_N be the testing sequence in which the elements are tested in the order in which they are numbered; and let S'_N be the testing sequence which differs from S_N in that two djoining elements r and s, $s \equiv r+1 \leq N$, are ermuted. We then have

$$f(S_N) = E(x) = \sum_{i=1}^{N}\left[\rho_i + \sum_{j=1}^{i}\tau_j\right]\frac{\frac{p_i}{q_i}}{\sum_{k=1}^{N}\frac{p_k}{q_k}};$$

$$\left(\sum_{k=1}^{N}\frac{p_k}{q_k}\right)f(S_N) = \sum_{i=1}^{N}\rho_i\frac{p_i}{q_i} + \sum_{i=1}^{N}\frac{p_i}{q_i}\sum_{j=1}^{i}\tau_j \tag{7}$$

and

$$\begin{aligned}\left(\sum_{k=1}^{N}\frac{p_k}{q_k}\right)f(S'_N) &= \sum_{i=1}^{N}\rho_i\frac{p_i}{q_i} + \sum_{i=1}^{N}\frac{p_i}{q_i}\sum_{j=1}^{i}\tau_j \\ &\quad - \frac{p_r}{q_r}\left(\sum_{j=1}^{s}\tau_j - \tau_s\right) + \frac{p_s}{q_s}\left(\sum_{j=1}^{s}\tau_j - \tau_r\right) \\ &\quad - \frac{p_s}{q_s}\left(\sum_{j=1}^{s}\tau_j\right) + \frac{p_r}{q_r}\left(\sum_{j=1}^{s}\tau_j\right) \\ &= \left(\sum_{k=1}^{N}\frac{p_k}{q_k}\right)f(S_N) + \frac{p_r}{q_r}\tau_s - \frac{p_s}{q_s}\tau_r \\ &= \left(\sum_{k=1}^{N}\frac{p_k}{q_k}\right)f(S_N) + \frac{p_r}{q_r}\frac{p_s}{q_s}\left[\frac{\tau_s}{q_s}q_s - \frac{\tau_r}{p_r}q_r\right].\end{aligned}$$

Since $r < s$, and the elements are numbered according to (1),

$$\frac{\tau_s}{p_s}q_s - \frac{\tau_r}{p_r}q_r \geq 0;$$

therefore

$$f(S'_N) \geq f(S_N),$$

thus showing that the expected cost, given by (7), is a minimum when the elements are numbered according to (1) and are tested in the sequence 1,2,...,N.

If we allow for testing "by elimination," we have

$$\left(\sum_{j=1}^{N}\frac{p_j}{q_j}\right)f(S_N) = \sum_{i=1}^{N}\frac{p_i}{q_i}\rho_i + \sum_{i=1}^{N}\frac{p_i}{q_i}\sum_{j=1}^{i}\tau_j - \frac{p_N}{q_N}\tau_N.$$

Following the same development as before, we find that the optimal test sequence S_k is 1,2,..., k-1,k+1,...,N,k, with the elements numbered according to (1) and k determined by (8):

$$\left.\begin{aligned} h(k) &= \min_{1\leq j\leq N}\{h(j)\}, \text{ where} \\ h(j) &= \frac{p_j}{q_j}\sum_{i=j}^{N}\tau_i - \tau_j\sum_{i=j}^{N}\frac{p_i}{q_i} - \frac{p_j}{q_j}\tau_j \end{aligned}\right\} . \quad (8)$$

The expected cost of this sequence is given by

$$\left(\sum_{j=1}^{N}\frac{p_j}{q_j}\right) f(S_k) = \sum_{i=1}^{N}\frac{p_i}{q_i}\rho_i + \sum_{i=1}^{N}\frac{p_i}{q_i}\sum_{j=1}^{i}\tau_j$$

$$+ \frac{p_k}{q_k}\sum_{j=k}^{N}\tau_j - \tau_k\sum_{j=k}^{N}\frac{p_j}{q_j} - \frac{p_k}{q_k}\tau_k .$$

C. Multilevel Equipment

Assume that the equipment is "modularized" at several "levels," i.e., that the equipment consists of N modules; that the mth module consists of i_m submodules, $m=1,2,\ldots,N$; that each module in turn consists of some number of sub-sub modules, etc. If, at each level, testing can be performed in accordance with our restrictions (either as stated in Section I, or as stated in Sections IV-A or IV-B, above) then we can use the above developed rules at each level, provided we adjust the "costs" accordingly, as follows.

When determining the sequence in which to test the modules, let T be the cost of an over-all test of the equipment, let τ_i be the cost of testing the ith module as a whole, and let ρ_i be the expected cost of testing and repairing the entire collection of submodules of the ith module. When determining the sequence in which to test the submodules of the ith module, let T be the cost of an over-all test of that module, let τ_j be the cost of testing the jth submodule of the ith module as a whole, and let ρ_j be the expected cost of testing and repairing the sub-sub modules of the jth submodule of the ith module, etc.

That the previously developed rules can be thus applied is an immediate consequence of Bellman's Principle of Optimality.[3]

V. INDETERMINATE TESTS

We have up to now considered tests whose only possible outcome is "pass" or "fail." In fact, it is possible that a test have an indeterminate outcome, neither pass nor fail, i.e., that the application of the test yields no information whatsoever. Let z_i be the probability that testing the ith item will result in an indeterminate outcome, and let a given test be applied repeatedly until it results in an unambiguous outcome. If θ_i is the total cost of testing the ith item under this regime, then

$$E(\theta_i) = \sum_{n=1}^{\infty}(n\tau_i)z_i^{n-1}(1-z_i) = \frac{\tau_i}{1-z_i} .$$

All the previously derived results hold under this regime provided that τ_i is replaced by $\tau_i/(1-z_i)$.

VI. ESTIMATION OF p_i

As stated in Section I,

$$p_i(t) = [F_i(t) - F_i(t_o)]/[1-F_i(t_o)]$$

for original elements, and

$$p_i(t) = [F_i(t-y_i) - F_i(t_o-y_i)]/[1-F_i(t_o-y_i)]$$

for replacement elements operating since the time y_i. Thus the estimation of p_i has as a prerequisite the estimation of the underlying failure distributions of the elements.

The estimation of the p_i is somewhat simplified if one assumes that 1) the elements fail in a random fashion, i.e., their failure distribution is the exponential distribution; and 2) the interval between examinations is constant, say w time units (following a diagnosis, the next examination is performed w time units after the end of diagnosis).

Then

$$p_i(t) = \frac{F_i(t) - F_i(t-w)}{1-F_i(t-w)} = \frac{1-e^{-t/\lambda_i} - [1-e^{-(t-w)/\lambda_i}]}{1-[1-e^{-(t-w)/\lambda_i}]}$$

$$= \frac{e^{-(t-w)/\lambda_i}[1-e^{-w/\lambda_i}]}{e^{-(t-w)/\lambda_i}} = 1-e^{-w/\lambda_i} ,$$

where λ_i is the mean life of the ith component.

Thus, under assumptions 1) and 2), and if λ_i is known, $p_i(t)$ can be calculated once and for all. If λ_i is not known, one can estimate p_i by

$$\hat{p}_i = \frac{n_i}{j} ,$$

[3] R. Bellman, "Dynamic Programming," Princeton University Press, Princeton, N.J.; 1957.

vhere n_i is the total number of times that ele-
ment i was found defective in all examinations,
ınd j is the total number of past examinations,
ncluding those in which the equipment was found
ood. Also, q_i is estimated by

$$\hat{q}_i = 1 - \frac{n_i}{j}.$$

In the "single failure" model, we indicated the
ossibility of not having to examine the equipment
eriodically. If that is the case, and if the ele-
nents are assumed to have the exponential failure
listribution, then

$$\hat{p}_i = 1-e^{-\gamma_i (t-t_0)}, \quad q_i = 1-p_i$$

vhere t_0 is the termination time of the last pre-
vious diagnosis, t is the time at which diagnosis
s about to be performed, and

$$\gamma_i = \frac{n_i}{L}$$

vith n_i being the number of times element i was
found defective and L being the total operating
ime of the equipment.

VII. AN EXAMPLE

An equipment consists of five elements A,B,C,
D and E. Each element is subject to random fail-
ıre, i.e., has an exponentially distributed life.
The equipment is put into operation at time zero,
s examined and found good at 1 hour, is examined
ınd found bad at 2 hours. Then

$$p_i = \frac{F_i(2) - F_i(1)}{1-F_i(1)} = \frac{1-e^{-2/\lambda_i}-(1-e^{-1/\lambda_i})}{e^{-1/\lambda_i}}$$

$$= \frac{e^{-1/\lambda_i}(1-e^{-1/\lambda_i})}{e^{-1/\lambda_i}} = 1-e^{-1/\lambda_i},$$

vhere λ_i is the mean life of element i, in hours.

Table I gives the known mean lifetimes of the
ive elements, the p_i calculated as above, and the testing costs τ_i (in arbitrary units); also, the values of $\tau_i q_i/p_i$, and the resultant ordering according to criterion (1).

TABLE I

Element	τ_i	λ_i	p_i	q_i	$\tau_i q_i/p_i$	order
A	0.72	1.80	0.43	0.57	0.95	5
B	0.68	1.44	0.50	0.50	0.68	1
C	0.95	1.30	0.54	0.46	0.81	4
D	0.73	1.42	0.50	0.50	0.73	2
E	0.85	1.32	0.53	0.47	0.75	3

If we follow the model of Section IV-a (i.e., include superfluous tests on the end), the optimal sequence is thus found to be B,D,E,C,A. If we follow the general model, criterion (5) must be used to select one of the five elements to be transferred to the end of the testing sequence.

VIII. COMMENT

A model similar to our general model has been treated by Johnson,[4] but it can be shown that his expression for $f(S_N)$ is too large by the factor $(1-Q_1)$. Johnson also treats the "single failure" model, but fails to indicate that he is treating that case in terms of conditional probabilities. Gluss[5] has treated the "single failure, no elimination" model, though in a manner different from ours.

IX. ACKNOWLEDGMENT

The author is indebted to Dr. R. E. Beckwith for his many helpful comments.

[4] S.M. Johnson, "Optimal Sequential Testing," RAND Corp., Santa Monica, Calif., Res. Memo. No. 1652; 1956.

[5] B. Gluss, "An optimum policy for detecting a fault in a complex system," Operations Res., vol. 7, no. 4; July-August, 1959.

New Autopsy Techniques for Transistors and Relays*

C.B. CLARK†, MEMBER, IRE, AND E.F. DUFFEK†

Summary—A new method for opening hermetically-sealed metal cases is described. Instead of mechanically sawing or cutting the metal, an electrochemical process is used. Two methods are described, static electrolysis (anodic dissolution) and jet electrolysis. Examples of the application of these methods to the "autopsy" of failed transistors with 6-mil Kovar shells and relays with 15-mil brass shells is shown.

INTRODUCTION

One of the jobs often done by a reliability group involves the examination of failed parts. In many cases a cursory visual examination is sufficient to determine if the component died a natural death or if its demise was violent. In the case of hermetically-sealed devices such as transistors and relays, however, external examination is not very helpful. It is possible to use X-ray methods to find out what happened inside the unit, but if the component is small, high-definition X ray is needed. This type of X-ray equipment is expensive and often unavailable.

It is much more informative to actually open the case (autopsy) to see what caused the failure. Opening a transistor case mechanically, however, often disturbs the rather delicate emitter and collector junction wires, particularly if the case is filled with "moose gunk."[1] The usual methods of cutting and grinding were tried, but pressure from the filling material disturbed the lead wires as the case was removed. Attempts were also made to dissolve the moose gunk away by admitting solvent through a hole in the transistor case, but this was not satisfactory.

The "autopsy" technique we have developed uses an electrochemical process to remove the case, leaving the internal transistor and relay elements intact. The method is very simple, involving only a holder for the transistor or relay, a shaped cathode, acid solution and a small dc power supply. Two procedures were used, a (static) electrolysis method for transistors and a jet electrolysis method for relay cans. The following description will indicate how these were carried out.

TRANSISTOR "AUTOPSY"

The failed transistors we were concerned with have a 170-mil OD case of about 6-mil-thick Kovar. The defective unit, held in a small clip, is positioned within a 7/16-inch-diameter hole punched in the stainless steel cathode as shown in Fig. 1.

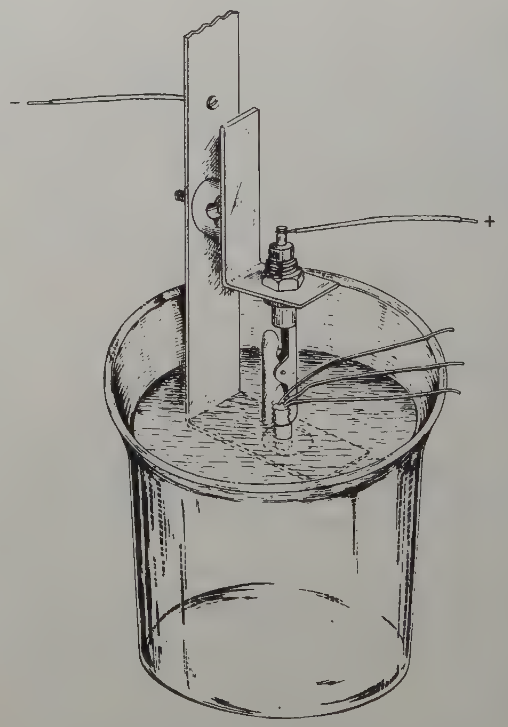

Fig. 1.—Static electrolysis equipment.

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[1]See Airlines Electronic Engineering Committee 1960 Letter No. 29 for origin of the term "moose gunk," referring to the mysterious mixture of silicone grease and other materials put inside the case by some transistor manufacturers.

small Alnico magnet is used to position the ransistor so that equal distances exist between lectrodes all around the transistor case. The ntire assembly is then immersed in a 20 per ent hydrocholoric acid solution at room tempera- ure. A current of about 1/4 ampere is applied rom a constant current supply, making the tran- istor case anodic with respect to the stainless teel holder. In 20 to 30 minutes the voltage cross the terminals, initially 4 volts, begins to ise, indicating that the 6-mil case has been dis- olved. The process taking place is essentially nodic dissolution (etching) of the Kovar. Hydro- hloric acid was chosen on the basis that Kovar vould dissolve rapidly only with the condition of pplied current. Thus, for example, a heavy gold late on a transistor may be more readily re- noved by anodic treatment in a sodium cyanide olution. Solutions to dissolve other metals should e considered similarly.

The unit is washed after removing it from the ath, and examined under a microscope. The noose gunk filling is sometimes clear enough so hat the nature of the fault is discernible. If the iller material is not clear enough to see through,

Fig. 2—Transistor with case removed, severe overload.

it can be dissolved in a vapor degreaser using ethylene dichoride at 30° to 35° C.

Two transistors opened by this technique and degreased are shown in Figs. 2 and 3. Fig. 2 clearly represents a case of severe overload, since the emitter wire has been melted into a ball. The reason for the failure of the unit shown in Fig. 3 is not clear, but it is definitely not a case of very severe overload as in the previous unit. The emitter junction is fractured, either by mechanical shock or possibly by thermal shock from a mild overload.

Fig. 3—Transistor with case removed, no severe overload.

RELAY "AUTOPSY"

One of the limitations of the static electrolysis method used on the transistors is nonuniform current distribution in the electrolyte. This is particularly troublesome when the can shape is noncircular, as in the crystal can relays.

To avoid difficulties with nonuniform current distribution, and allow higher etching currents,

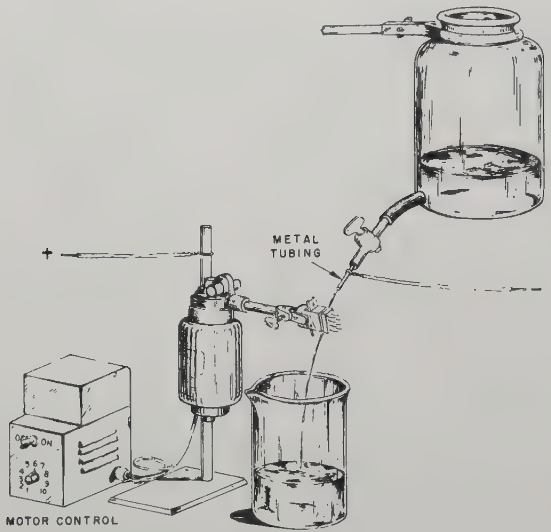

Fig. 4—Jet electrolysis equipment.

the equipment shown in Fig. 4 was used.

The relay to be opened is clamped in a rotatable fixture as shown. The paint is scratched off the relay case where the cut was wanted. A stream or jet of 10 per cent HCl solution is directed at the metal relay case through a metal tubing, and a 24-volt supply connected between the metal fixture and the metal tubing, tubing negative, and relay case anodic. The motor speed control is adjusted to operate the motor at about 20 rpm. A current of about 1 ampere resulted in the case being cut in two in 20 to 30 minutes. About 1 liter of solution was made up and re-used several times by pouring it back into the upper container.

A picture of a relay opened by the above procedure is shown in Fig. 5. Note the clean, well-defined cut. The case is made of brass, and is about 15 mils thick. Although some of the dilute acid reached the interior of the relay, little damage was done.

The jet electrolysis method should be useful on many items, since it is fast and adaptable to different shapes. It would probably be useful for opening transistors as well, particularly if the case is not round.

Fig. 5—Relay with cover removed by jet electrolysis.

CONCLUSION

The "autopsy" techniques presented above were developed for examination of internal transistor and relay elements. The method is fast, inexpensive and adaptable to circular or irregular shaped components and should aid those engaged in reliability and quality control evaluations, especially those involving life tests. Although we have used this cutting method only on Kovar and brass cans, it should be adaptable to other metals without much difficulty.

On Prediction of System Behavior*

JOAN R. ROSENBLATT†

INTRODUCTION

The subject of this paper is, in general, the roblem of putting together a prediction concern- ng the way in which a complex system will func- ion. Particular emphasis is given to a discussion of some of the types of definitions and choices vhich are made before such a prediction can be ssembled.

A theoretical discussion of prediction of system erformance deals with 1) a variable or a set of ariables by which system performance is to be ssessed, and 2) variables describing the parts of he system. The dependence of system perform- nce on properties of parts of the system is as- umed to be given by a mathematical description f the relations among these two types of vari- bles. In the first part of this paper, a quite gen- ral formal statement of the problem is given. In he two further parts of the paper, illustrations re given suggesting some of the consequences of n explicit attempt to realize this formal approach.

In the second part of the paper, some familiar imple mathematical models for prediction are tated as special cases of the general formulation. ome illustrations indicate the possibilities for enriching" these simple models.

In the third part of the paper, some possible onsequences of explicit attention to problems of he definition of variables are developed, by means f simple illustrations involving the treatment of a nixture of two modes of failure.

A GENERAL STATEMENT OF THE PREDICTION PROBLEM

Consider first the set of variables representing roperties of the system by which its performance s assessed at time t. We may call this the sys- em variable (in general a vector variable) and enote it by $x_0(t)$. Among the system properties hich might be represented in $x_0(t)$ are the values of system outputs (where appropriate), the age of the system in terms of hours of active use, and the number of surviving redundant components. In addition, it may be appropriate for $x_0(t)$ to represent the presence or absence of attributes of the system: "operating" vs "failed," "on" vs "off," "in storage" vs "in use."

The value or probability distribution of $x_0(t)$ determines the value of some "figure of merit" by which reliability is to be measured.

Consider next a set of variables $x_1(t)$, $x_2(t)$, . . . , $x_n(t)$, which represent factors determining system performance at time t. Environmental factors and human operators may be included among these; for convenience, they will be called subsystem variables. A subsystem variable represents the properties of a subsystem which are sufficient to describe its effect on system performance at time t.

The specification of a set of subsystem variables involves, first, the choice of the number of subsystems which is considered appropriate (or feasible) for representation of system performance. To complete the specification of a set of subsystem variables, it is necessary to state how their values or joint probability distribution may be determined or estimated.

Consider now the mathematical description of the dependence of system performance on properties of the subsystems. This will, in general, have two aspects. First, the value of the system variable $x_0(t)$ may be at least in part directly determined by the values of some of the subsystem variables. This functional relation may be denoted formally by

$$x_0(t) = F\{x_1(t), \ldots, x_n(t); t\} .$$

Second, the description of dependence includes a statement of the set of assumptions by which the probability distribution of $x_0(t)$ is determined. These assumptions specify properties of the joint distribution of the subsystem variables and the manner in which the values and distribution of the subsystem variables affect the distribution of $x_0(t)$.

Finally, consider briefly the role of the time variable t in this general statement. It is evident

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Reprinted from Proceedings of the New York Univer- ity—Industry Conference on Reliability Theory, rdsley-on-Hudson, N.Y., June 9-11, 1958.
National Bureau of Standards, Washington, D.C.

that in an operational prediction problem, this should be real time, with reference to a well-defined "zero" point which is the same for each subsystem.

The variable t is included explicitly as an argument in the relation represented by F, because the form of the relation may be different at different times (e.g., storage vs operating times).

The explicit presence of t in connection with each of the subsystem variables calls attention to the possible presence (and representation) of sequential or cyclic time-phasing in the order of use of parts of the system. It may also be necessary or useful to account for "parts" of the system which are effectively present during some time intervals but not during others; for example, protective packing for storage, or the rocket itself in a rocket-launching system.

Selection of appropriate variables and assumptions provides the framework, the mathematical model, for prediction. Given this framework, the process of prediction consists of assembling information about the subsystem variables, evaluating the system variable or its probability distribution, and then calculating the values of some criteria of satisfactory performance, or figures of merit, which are functions of the system variable and its probability distribution.

It is obvious, however, that the selection of the framework for prediction is an intrinsic and fundamental part of the prediction process. It requires detailed specification or definition of each of the subsystem variables and of the system variable, and includes the choice of the form of the functional relation F and the choice of assumptions by which the distribution of $x_0(t)$ is determined.

These definitions and choices will always be made, at least implicitly. The purpose of this paper is to examine some of the possible consequences and advantages of early explicit recognition and investigation of the available choices.

SIMPLE PREDICTION MODELS

In this section of the paper, we note some of the types of choices which are made in using a model based on the "product rule." Some obvious possibilities for enrichment of this simple approach will emerge.

The system variable is defined by

$$x_0(t) = \begin{cases} 1 & \text{if the system "is working" at time } t, \\ 0 & \text{otherwise.} \end{cases}$$

The performance criterion of interest is the probability that the system will survive through the time interval (0,t). Let

$$p_0(t) = \Pr[X_0(\tau) = 1, \quad 0 \leq \tau \leq t].$$

The subsystem variables are similarly defined:

$$x_i(t) = \begin{cases} 1 & \text{if the ith subsystem "is working" at time } t, \\ 0 & \text{otherwise,} \end{cases} \qquad i=1,2,\ldots,n.$$

The relation of $x_0(t)$ to the subsystem variables is specified by the assertion that the system "is working" if and only if each of the subsystems is

$$x_0(t) = x_1(t) \cdot x_2(t) \ldots x_n(t).$$

Now it is assumed that each of the subsystem variables is statistically independent of the others. Under this assumption,

$$p_0(t) = p_1(t) \cdot p_2(t) \ldots p_n(t),$$

where

$$p_i(t) = \Pr[X_i(\tau) = 1, \quad 0 \leq \tau \leq t].$$

If the additional assumption is made that the subsystem lifetimes are exponentially distributed, we obtain

$$p_0(t) = \exp\{-\lambda_1 + \lambda_2 + \ldots + \lambda_n)t\},$$

where λ_i is the failure rate of the ith subsystem.

By means of various kinds of modifications of the subsystem survival probabilities $p_i(t)$, this simple model based on the assumption of independence among subsystems may be generalized in various ways. Suppose, for example, that the system is to be stored for a period of time t_s before it is used. Letting t=0 at the beginning of the storage period, we might have at time $t > t_s$,

$$p_i(t) = q_i(t_s) \times q_i'(t - t_s),$$

where $q_i(t_s)$ is the probability that the ith subsystem survives under storage conditions through the period $(0,t_s)$, and $q_i'(\tau)$ is the (conditional) probability that the ith subsystem survives through an operating interval of length τ after surviving a storage period. In a more complicated representation, the function $q_i'(\tau)$ could be made to depend on t_s.

As another example, consider the possible resence in a system of an "initial use" subsys-em, which is required to survive through the time nterval $(0,t^*)$ but is not needed thereafter. Thus,

$$x_i(t) = \begin{cases} 1 & \text{if the subsystem is working at time } t, \text{ or if } t > t^* \\ 0 & \text{otherwise;} \end{cases}$$

$q_i(t)$ denotes the survival probability of the sub-ystem, then

$$p_i(t) = \begin{cases} q_i(t) & \text{if} \quad t \leq t^* \\ q_i(t^*) & \text{if} \quad t > t^*. \end{cases}$$

Another approach to generalization may be nade through the explicit representation of (ex-ernally caused) environmental stresses, in such way that the subsystems may be assumed to be onditionally independent, given the environmental onditions. One representation of this type [2] is ne following. Suppose there is a "stress level" hich may be either "critical" or "not critical," uch that the system can work only if it is not ritical. Let

$$x_e(t) = \begin{cases} 1 & \text{if environmental stress is not critical at time } t \\ 0 & \text{if it is critical,} \end{cases}$$

nd

$$p_e(t) = \Pr[X_e(\tau) = 1, \quad 0 \leq \tau \leq t].$$

ow

$$x_0(t) = x_e(t) \cdot x_1(t) \dots x_n(t)$$

nd, if we interpret $p_i(t)$ as the conditional survi-al probability of the ith component when noncriti-al conditions prevail, then

$$p_0(t) = p_e(t) \cdot p_1(t) \dots p_n(t).$$

A second representation involving the occur-ence of environmental stresses is the following. ppose there were random (i.e., Poisson) oc-rrences of "shocks" such that each occurrence oduces changes in the forms of the survival obability functions for some or all subsystems. or instance, there might be random occurrences a certain type of electrical transient, produced one subsystem and causing degradation of the survival probabilities of several others. Suppose that the probability of continued survival of a sub-system at time t depended only on the total num-ber of "shocks" sustained, and the time elapsed since the most recent one. Thus, if k "shocks" have occurred at times $\tau_1, \tau_2, \dots, \tau_k$ $(k=0,1,2,\dots,\tau_0 = 0)$, the (conditional) survival probability of the ith subsystem may be written

$$p_i^{(k)}(t) = p_{i0}(\tau_1)\, p_{i1}(\tau_2 - \tau_1) \dots p_{ik}(t - \tau_k),$$

$0 < \tau_1 < \tau_2 < \dots < \tau_k < t$. The conditional survival probability for the system, given that k "shocks" have occurred in the interval $(0,t)$, is then

$$p_0^{(k)}(t) = k! \int_{0<\tau_1<\dots<\tau_k<t} \cdots \int \prod_{i=1}^{n} p_i^{(k)}(t)\, d\tau_1 \dots d\tau_k,$$

and

$$p_0(t) = \sum_{k=0}^{\infty} e^{-\lambda t} \frac{\lambda^k}{k!} p_0^{(k)}(t),$$

where λ is the rate of occurrence of the "shocks." Whether or not $p_0(t)$ can be explicitly calculated depends on the forms of the conditional survival probability functions for subsystems.

A product-rule formalism may have serious disadvantages when the specification of subsystems and time relationships is implicit and incomplete. The object of the foregoing discussion is to indi-cate how the approach may be enriched so that in-dependence is assumed where independence is plausible, or where a conditional independence mechanism is reasonable.

CLASSIFICATION OF FAILURES

In this section, we consider another aspect of model construction, namely, the impact of the classification of failures on the model form and model testing. The discussion is conducted in terms of a set of simple illustrative calculations.

Consider an equipment for which the conditions of use are as follows. It is used regularly (daily, weekly, etc.), or essentially so, in an activity which we may call a "mission." It is "turned on" at the beginning of the mission, is in use throughout the mission, and is "turned off" at the end. The dura-tions of the missions may or may not be essentially constant. An example of such an equipment would be an airborne electronic instrument.

The question arises whether the lifetime of a part of such an equipment should be measured (and predicted) in terms of use—hours (total flying-time, say) or in terms of the number of uses (total number of missions flown). The time between missions will be ignored in this discussion.

Suppose that the underlying life distribution for the equipment has the following form:

$$\Pr(\text{survive a mission of duration } t) = \rho e^{-\lambda t},$$

where

ρ = probability of surviving "turn-on" (including, e.g., electrical transients, operator errors)

$e^{-\lambda t}$ = probability of surviving to time t on condition that no turn-on failure occurs, where λ is the (conditional) failure rate.

Suppose certain life-testing data were available from acceptance inspection tests, for evaluating the "reliability" of the equipment. In particular, suppose there were a fixed time on test c, and that test results consisted only of the proportion surviving. These data could be interpreted in accord with either of the two limiting cases of the model stated above. For simplicity, sampling variation will be ignored, and the observed proportion surviving will be assumed to be equal to the survival probability.

Number-of-Missions Interpretation

It is assumed that there is a probability

$$\gamma = \Pr(\text{survive one mission}),$$

independent of the length of the mission. In fact, the observed proportion is

$$\hat{\gamma} = \rho e^{-\lambda c}.$$

Number-of-Hours Interpretation

It is assumed that there is a constant failure rate μ. In fact, the observed proportion is

$$e^{-\hat{\mu} c} = \rho e^{-\lambda c},$$

whence

$$\hat{\mu} = \lambda + \frac{1}{c} \ln \frac{1}{\rho}.$$

We turn now to an examination of the consequences of employing one or the other of these two interpretations, in predicting the "reliability" of the equipment.

Case 1—Missions of Constant Duration δ

It is desired to predict the probability that the equipment will survive m missions. Thus, the quantity to be predicted is

$$P_0^m = (\rho e^{-\lambda \delta})^m.$$

Consider the two predictors corresponding to the two interpretations stated above:

$$P_I^m = \hat{\gamma}^m \qquad \text{(Prediction I)}$$

$$P_{II}^m = e^{-\hat{\mu} m \delta} \qquad \text{(Prediction II)}.$$

Comparison of each of these with P_0^m may be made in terms of the ratios

$$P_I/P_0 = e^{-\lambda(c-\delta)},$$

$$P_{II}/P_0 = \rho^{(\delta-c)/c}.$$

If the actual mission duration δ is less than the test duration c, then Prediction I is pessimistic while Prediction II is optimistic. On the other hand, if $c < \delta$, Prediction I is optimistic while II is pessimistic. Specifically, e.g.,

$$P_I/P_0 < 1 \quad \text{if} \quad \delta < c;$$

Prediction I is pessimistic in the sense that it understates the "true" survival probability.

Now, if $c = \delta$, both predictions are the same and correct. They are, in effect, "nonparametric" relative to this model.

It may be remarked that the calculations for this case suggest that if the relative weights of the two modes of failure are unknown (both being possibly present), then estimation based on $c = \delta$ (test time = mission duration) would be desirable. More generally, an investigation of a model representing several possible modes of failure may lead to the selection of an experimental procedure and prediction technique which are not too sensitive to errors in assumptions about modes of failure.

Case 2—Missions of Mean Duration δ

Now suppose that missions are of variable duration, but that the duration D has a known probability distribution with mean δ. It is desired to

redict the probability that the equipment will survive m missions, i.e., the quantity

$$Q_0^m = \rho^m (E\, e^{-D\lambda})^m,$$

here E is the expectation operator and $ED = \delta$. or illustrative purposes, some comparisons of redictors will be calculated for the case where D as the exponential distribution. For this special ase,

$$Q_0^m = \left(\frac{\rho}{1+\lambda\delta}\right)^m .$$

Employing the number-of-missions interpretaon, one would again use Prediction I. Now

$$P_I/Q_0 = (1 + \lambda\delta)e^{-\lambda c}$$
$$= 1 + \lambda(\delta - c) + 0(\lambda^2).$$

hus, in almost the same way as in Case 1, Preiction I is pessimistic if $\delta \leq c$. If $\delta > c$ and $\delta - c$) is sufficiently large relative to λ, then rediction I is optimistic.

When the number-of-hours interpretation is mployed, two approaches are possible. First, rediction II may be used as an approximation gnoring variability of mission duration). We ave

$$P_{II}/Q_0 = (1 + \lambda\delta)e^{-\lambda\delta}\rho^{(\delta-c)/c} .$$

ow $(1 + \lambda\delta)e^{-\lambda\delta} < 1$ so that Prediction II is essimistic if $\delta \geq c$, again as in Case 1. In order at P_{II} provide an optimistic prediction for ase 2, it would be necessary that $c >> \delta$.

A second approach within the number-of-hours terpretation is based on the experimenter's nowledge of the distribution of the variable mision duration D. We introduce a third predictor r the probability of surviving m missions:

$$P_{III}^m = (Ee^{-\hat{\mu}D})^m = (1 + \hat{\mu}\delta)^{-m} \quad \text{(Prediction III)}.$$

onsider

$$P_{III}/Q_0 = \frac{1}{\rho}\left[1 + \frac{\delta}{c(1+\lambda\delta)} \ln \frac{1}{\rho}\right]^{-1} .$$

is easily verified that

$$P_{III}/Q_0 > 1 \quad \text{if} \quad \delta = c,$$

e., that P_{III} tends to be more optimistic than P_{II} (which was noted to be generally pessimistic).

Calculations for this simple model have illustrated the possibilities for qualitatively appraising different biases of prediction in the event that there are two modes of failure with unknown relative importance.

If the bias can be serious, it may be desirable to alter the form of the testing or experimental procedure used to obtain data.

A recent paper by Stoller [3] treats some statistical issues which would arise in considering the particular model used as an illustration in this section.

CONCLUSION

In the context of a general representation of models of system performance, attention has been drawn to some basic methodological considerations which arise in the composition of prediction models for complex systems. In particular, we have examined the consequence of certain types of model or definition choices: first, choices concerning the relative independence or interdependence of parts in the elaboration of a simple model; second, choices concerning the classification of modes of failure. These are basic elementary types of choices, and some aspects have been treated explicitly, as they must be treated in an operational approach to prediction problems for specific complex assemblies. Such choices are explicitly realized in the form of the data collection or experimental program (including possibly some simulation experiments).

An alternative statement of the methodological point of this paper may be made from the standpoint of the reliability statistician or engineer. The relevance and meaningfulness of reliability predictions are in large measure already determined by the form and fine structure of the subsystem or component data. In the context of an explicit model, we may expose and examine the definite methodological choices available to the reliability statistician or engineer for controlling the form of these data.

REFERENCES

Many persons have discussed particular aspects of the methodological issues considered in this paper. An illustrative reference is the section, "Measurement of Time for Reliability Evaluation" (Chapter III, Section 3.1, page 21 of [1]). An exhaustive list of references has not been compiled.

[1] Aeronautical Radio, Inc., "Concepts and Tentative Techniques for Reliability Assurance," Progress Rept. No. 1, Air Force Reliability Assurance Program; February 15, 1956.

[2] J.R. Rosenblatt, "On prediction of system performance from information on component performance," Proc. WJCC, pp. 85-94; February, 1957.

[3] D.S. Stoller, "A failure model for equipments undergoing complex operation," Operations Res., vol. 6, pp. 723-728; 1958.

An Application of the Information Theory Approach to Failure Diagnosis*

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Summary—Brulé, Johnson, and Kletsky[1] have eveloped a technique based on information theory hich leads to highly efficient procedures for diag- osing equipment failures. This paper demon- rates by means of a practical example the valid- y of this technique. In addition, the feasibility of e approach is shown and the procedure to be used its implementation is outlined in detail. The aper concludes with a general discussion includ- g comments on the generality of the technique, e possibility of machine computation, and possi- e areas of application.

I. INTRODUCTION

As the complexity of newly-developed high- erformance systems continues to grow, the asso- ated problem of maintaining and repairing these ystems becomes increasingly important. De- ands for higher reliability, longer life, and orter periods of down time are factors which ork to increase the severity of the maintenance oblem.

The problem has been attacked on several onts. Manufacturers continue to improve the liability of component parts, preventive main- nance procedures tend to reduce over-all down me, and careful mechanical and electrical design mplify repairs.

One area of the over-all maintenance problem ich has been neglected until recently is that of ult location. Yet, field experience indicates that is area accounts for as much as a third of the tal down time on the equipment. It appears asonable therefore that an examination of the sic fundamentals of diagnostic procedures could bstantially reduce this significant portion of wn time.

Brulé, Johnson, and Kletsky have studied the problem of equipment diagnosis and have developed a technique based on information theory which leads, in theory at least, to highly efficient diagnostic procedures.[1,2] The main purpose of the work to follow is to demonstrate the validity of this technique. To this end, a diagnostic procedure for a relatively simple communications receiver has been developed using the proposed technique. For receivers of this general type, we know what an efficient diagnostic procedure should look like as a result of studying the trouble-shooting methods employed by highly competent technicians with wide experience in receiver repair. The results show that the information theory technique yields a diagnostic procedure which is not essentially different from that which would be used by a competent technician.

As a result of the demonstrated usefulness of the technique, we suggest in Section VII that it will also prove useful in the development of self-monitoring machines, in the design of new systems, and in helping alleviate the maintenance problem associated with existing systems.

II. TECHNIQUE

The information theory approach can be considered as a formalization of the trouble-shooting techniques employed by an expert technician. In diagnosing an equipment failure, the technician has available a large number of tests. He is able to choose from these a set which is sufficient to diagnose the equipment. To each of these tests he assigns a cost which may be dependent on those tests previously performed. When a test is performed, the technician learns something about the equipment; that is, each test removes a certain amount of ambiguity concerning the location of the fault.

The problem is to find a sequence of tests which will diagnose the equipment in an efficient fashion, that is, at minimum cost. Johnson[2] has shown that

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lec. Engrg. Dept., Syracuse University, Syracuse, Y.

.D. Brulé, R.A. Johnson, and E.J. Kletsky, "Diagnosis equipment failures," IRE Trans. on Reliability and ality Control, vol. RQC-9, pp. 23-34; April, 1960.

[2] R.A. Johnson, "An information theory approach to diagnosis," Proc. Sixth Natl. Symp. on Reliability and Quality Control, Washington, D.C.; January 11-13, 1960.

a highly efficient sequence of tests can be assured by choosing the tests on the basis of a figure of merit derived from information theory.

The figure of merit to be used is the ratio of ambiguity removed by a test to the cost of performing the test. That is

$$F_k = \frac{\overline{\Delta A_k}}{C_k} = \frac{-P \log_2 P - (1-P)\log_2(1-P)}{C_k}$$

where P is the a priori probability that the test will pass and C_k is the cost of performing the test. An efficient sequential testing diagram can be constructed using this figure of merit according to the following schedule:

1) Evaluate F_k for each of the possible tests.
2) Choose the test with the highest F_k.
3) Alter the cost of performing the remaining tests on the basis of having performed this and other tests previously.
4) Alter the a priori probability of passing for each of the remaining tests on the basis of knowledge gained by having performed this and other tests previously.
5) Repeat the procedure until the entire sequential diagram is determined.

III. APPLICATION

In order to demonstrate the use of the above schedule, a standard Air Force communication receiver (R-278B/GR) has been analyzed and a diagnostic procedure prepared. The receiver can be represented by the elementary equipment diagram shown in Fig. 1. This diagram represents the interactions between the power supply, mechanical tuning system, and signal circuits which are necessary to provide a useful output. Also shown are the required primary stimuli (antenna signal, primary power, mechanical tuning). The assumption has been made that the mechanical tuning system is in operating condition and, hence, failures can only occur either in the power supply or signal circuits. It is further assumed that interconnecting cables are in operating condition. It must be pointed out however, that these assumptions have been made only in the interest of simplicity and that the procedure is inherently capable of handling the unrestricted problem as well.

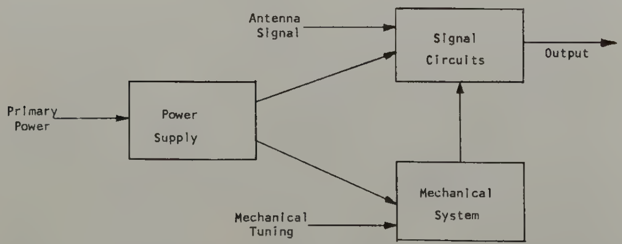

Fig. 1—Elementary equipment diagram.

Power Supply Analysis

Study of the receiver circuit diagram allows construction of an equipment diagram for the power supply, as shown in Fig. 2. Each box represents a functional element to which a failure may be attributed. Lines entering a functional element represent input stimuli (electrical, mechanical, or other) which must be present before the element is capable of providing an output. An output will be found if and only if the necessary stimuli are present and the functional element is good.

The equipment diagram provides the means by which an exhaustive list of possible tests can be formed. A test may consist of supplying all the necessary stimuli to an element and observing the response of this element. (For example, Rectifier 1 can be tested by supplying 115 volts ac and observing, as a response, the presence of the "raw B+" signal.) Or, tests may consist of supplying stimuli at the input of a group (cascade and/or parallel) of elements and observing a single output response. In general, the number of possible tests is exceedingly large. It is desirable to be able to reduce this number to more manageable proportions. This can be done by considering the cost associated with performing a test.

Test Cost Analysis

The cost of performing a test at a particular time is a function of many parameters. The most important of these are listed below:

1) Test equipment required.
2) Present state of disassembly of equipment under test.
3) Additional disassembly of equipment required.
4) Cost of supplying external signals.
5) Cost of actual test performance.

It is seen that the cost of performing a test is dependent on what tests have already been performed. Any method used to find an efficient testing procedure must consider this point.

Each test consists of a sequence or set of "unit operations," each of which is assigned a cost. The test cost is the sum of the individual unit operation costs. Unit operations can be either of fixed cost or of variable (decreasing) cost. Once a test

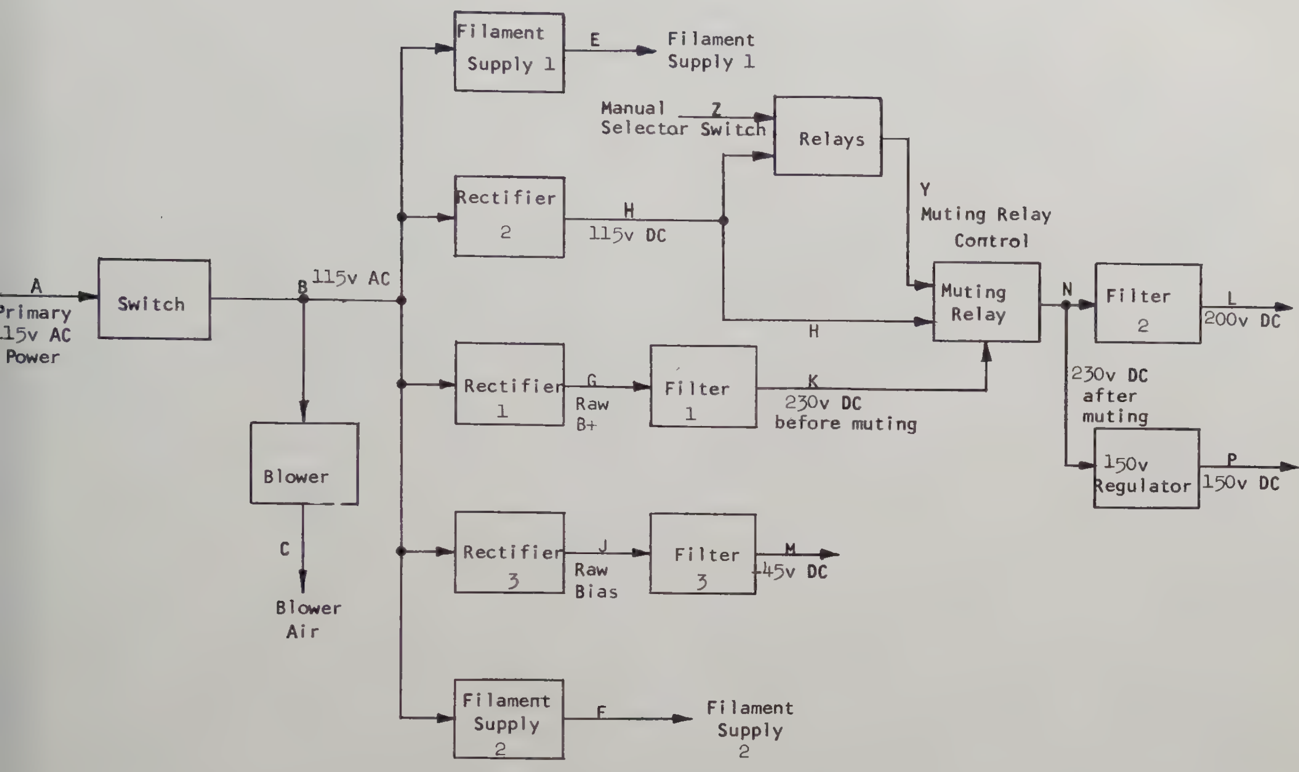

Fig. 2—Power supply equipment diagram.

s selected, those operations of variable cost as-ociated with the test are appropriately modified. Hence, subsequent tests requiring unit operations which have been previously used will, in general, have lower costs.

Unfortunately, data do not exist which allow an xact determination of unit operation costs. Reasonable estimates have been made based on xamination of the equipment under test, availa-ility of test equipment, time required to actually erform the test in the laboratory, etc. These stimates have been reduced to equivalent costs xpressed in man-hours. The number of digits arried in the test cost reflects the wide range of osts associated with the unit operations. It is oped that eventually, better time and cost studies vill have been made which will allow more pre-ise cost estimates.

Using the cost analysis as a guide, a subset of he large number of available tests can be chosen s reasonable tests to perform. This group of ests is shown in Table I, along with the initial ost of performing the test. Each test is desig-ated by an N-digit binary number where N is the umber of functional elements. (N = 13 for the ower supply.) The test number carries a "zero" in every position corresponding to an element which must be "good" in order that the test pass. In general, the test designation asks the question: "Are all of the elements i, j, . . . , k good?", where i, j, . . . k specify those elements which have a zero in the test designation. The binary test designations are easily determined from the equipment diagram. The alphabetic symbols for signals in Table I refer to those shown in Fig. 2.

For reference, a detailed description of a selected set of these tests is shown in the Appendix.

Probability Analysis

In addition to a suitable subset of tests, the application of the information theory technique requires that the a priori probability of failure of each functional element be known. An estimate of these probabilities can be made using the raw data from RCA investigations[3] of the R-278B/GR Receiver. Table II shows part quantities and corresponding failure rates for each of the functional elements of the power supply. (Failure rates

[3]"A Prediction of AN/GRC-27 Reliability," RCA Service Co., Inc., RADC-TN-58-18.

TABLE I

POWER SUPPLY TESTS

Test No.	Test of Signal	Switch	Blower	Filament 1	Filament 2	Rectifier 1	Rectifier 2	Rectifier 3	Filter 1	Filter 2	Filter 3	Relays	Muting Relay	150-volt Regulator	Signals Required	Initial Test Cost	Initial Probability of Passing
1	B	0	1	1	1	1	1	1	1	1	1	1	1	1	A	0.0201	.995
2	C	1	0	1	1	1	1	1	1	1	1	1	1	1	B	0.410	.988
3	E	1	1	0	1	1	1	1	1	1	1	1	1	1	B	0.275	.997
4	F	1	1	1	0	1	1	1	1	1	1	1	1	1	B	0.275	.997
5	G	1	1	1	1	0	1	1	1	1	1	1	1	1	B	0.275	.745
6	H	1	1	1	1	1	0	1	1	1	1	1	1	1	B	0.275	.745
7	J	1	1	1	1	1	1	0	1	1	1	1	1	1	B	0.275	.638
8	K	1	1	1	1	1	1	1	0	1	1	1	1	1	G	1.525	.985
9	L	1	1	1	1	1	1	1	1	0	1	1	1	1	N	1.525	.990
10	M	1	1	1	1	1	1	1	1	1	0	1	1	1	J	1.525	.989
11	N	1	1	1	1	1	1	1	1	1	1	1	0	1	YHK	3.055	.999
12	P	1	1	1	1	1	1	1	1	1	1	1	1	0	N	∞	.940
13	Y	1	1	1	1	1	1	1	1	1	1	0	1	1	ZH	1.530	.993
14	C	0	0	1	1	1	1	1	1	1	1	1	1	1	A	0.0001	.983
15	E	0	1	0	1	1	1	1	1	1	1	1	1	1	A	0.0102	.992
16	F	0	1	1	0	1	1	1	1	1	1	1	1	1	A	0.0102	.992
17	G	0	1	1	1	0	1	1	1	1	1	1	1	1	A	0.0201	.740
18	H	0	1	1	1	1	0	1	1	1	1	1	1	1	A	0.0201	.740
19	J	0	1	1	1	1	1	0	1	1	1	1	1	1	A	0.0251	.633
20	K	0	1	1	1	0	1	1	0	1	1	1	1	1	A	0.0201	.725
21	M	0	1	1	1	1	1	0	1	1	0	1	1	1	A	0.0201	.622
22	N	0	1	1	1	1	0	1	1	1	1	1	0	1	YKA	1.5501	.738
23	N	0	1	1	1	1	0	1	1	1	1	0	0	1	ZKA	1.5351	.731
24	N	0	1	1	1	0	0	1	0	1	1	0	0	1	ZA	0.0301	.461
25	L	0	1	1	1	0	0	1	0	0	1	0	0	1	ZA	0.0251	.451
26	P	0	1	1	1	0	0	1	0	1	1	0	0	0	ZA	0.0301	.401
27	Y	0	1	1	1	1	0	1	1	1	1	0	1	1	ZA	0.0251	.733

TABLE II

COMPUTATION OF FAILURE PROBABILITIES FOR POWER SUPPLY

	Switch	Blower	Filament No. 1	Filament No. 2	Rectifier No. 1	Rectifier No. 2	Rectifier No. 3	Filter No. 1	Filter No. 2	Filter No. 3	Relays	Muting Relay	150-volt Regulator
apacitors No.		1						3	2	2			
otal Rate		10						30	20	20			
elays No.											3	1	
otal Rate											90	30	
oils No.								1	1				
otal Rate								100	100				
esistors No.								1		2			1
otal Rate								60		100			175
witches No.	2												
otal Rate	60												
ransformers No.			1	1	1	1	1						
otal Rate			35	35	100	100	100						
ubes No.					1	1	1						1
otal Rate					3000	3000	4320						560
eaters No.							1						
otal Rate							45						
lower No.		1											
otal Rate		140											
otal Failure Rate	60	150	35	35	3100	3100	4465	190	120	130	90	30	735
robability of Failure	0.005	0.012	0.003	0.003	0.255	0.255	0.362	0.015	0.010	0.011	0.007	0.002	0.060

hown are 1000 times the failure rate expressed n per cent/1000 hours.) The a priori probability of failure of a given functional element is found by lividing its failure rate by the failure rate of the entire power supply. The procedure used here is not unlike that used by Ryerson and others in estimating failure probabilities.[4]

Using the derived a priori probabilities of failure, it is possible to compute the probability with which each of the tests in Table I will pass. This s done by summing the nonzero probabilities of the test over all the elements not known to be good in the previous state. (For example, $P_{T_{26}} = 0.012 + 0.003 + 0.003 + 0.362 + 0.010 + 0.011 = 0.401$.) The result of this operation is shown in the appropriate column of Table I.

Formation of the Testing Diagram

All the data required for the application of the figure of merit, F_k, have now been compiled. The figure of merit for each test can be computed in a straightforward manner. It should be emphasized that many tests can be thrown out prior to calculation by noting that high cost and/or low $\overline{\Delta A_k}$ lead

C.M. Ryerson, "RCA Reliability Program and Long ange Objectives," RCA, Camden, N.J.; March 15, 1955.

to low F_k. The test with the highest F_k is chosen as the first test in the testing diagram. From this test the two following states are constructed. The manner in which these states are determined is best illustrated by reference to Fig. 3.

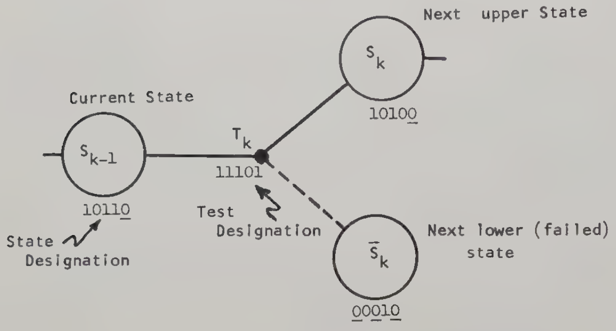

Fig. 3—Determination of states following a test.

S_{k-1} represents the state of the equipment prior to performing the test T_k. This state is given by an N-digit number containing only the digits 1, 0, and $\underline{0}$. There is a 1 in each position corresponding to elements not yet tested. There is a 0 in each position corresponding to elements known to be good. There is a $\underline{0}$ in each position corresponding to elements inferred to be good on the basis of a previously failed test. In the initial state, there are 1's in all positions since none of the elements have been tested.

S_k represents the state of the equipment if test T_k passes. This state is computed by multiplying S_{k-1} and T_k digit by digit without carry. $\bar{S}_k$ represents the state of the equipment if test T_k fails. It is computed by multiplying S_{k-1} and the complement of T_k digit by digit without carry. The 0's in T_k complement must be replaced by $\underline{0}$'s in order to prevent improper diagnosis if more than one element has failed.

Following the selection of a test, the cost of performing the succeeding tests must be modified on the basis of the test or tests already performed. This is done by altering the cost of unit operations which are common to unit operations used in previously performed tests. (For example, when Test T_{18} has been performed, the subsequent performance of Test T_{24} has a cost of only 0.020 since unit operations 26 and 28 result in no additional cost.)

Once a test is chosen and performed, the relative probabilities of element failure change. It is thus necessary to recompute the probability with which each of the possible succeeding tests will pass. As in the initial state, the new probabilities are found by summing the nonzero probabilities of the test over all the elements not known to be good in the previous state.

Additional tests in the testing diagram are determined by repeated application of the above procedures. If any state contains a single 1, that particular branch of the diagram is completed, since the element corresponding to the 1 has been isolated. The testing diagram is complete when every branch terminates in a state containing a single 1.

The resulting testing diagram for the power supply is shown in Fig. 4. Also shown are the costs of performing the tests and the a priori probability of failure of each functional element. Note that it is also possible to compute the average cost of locating a fault in the power supply. This is given by

$$\text{average cost} = \bar{C} = \sum_{j=1}^{N} \ell_j p_j + c_k p_k.$$

p_j = initial a priori probability of failure of jth element
ℓ_j = sum of test costs required to reach jth element
N = number of elements
c_k = cost of verifying test
p_k = probability that verifying test must be performed.

The verifying test must be used to determine unequivocally the state of the element isolated at the extreme end of the upper branch in the testing diagram. This element has been isolated only by inference. The verifying test to be used is the cheapest one which explicitly tests this final element.

The testing diagram shown in Fig. 4, in conjunction with the test descriptions of the type found in the Appendix, is sufficient to perform a diagnosis of the power supply.

Exactly the same procedure is used to prepare a testing diagram for the signal circuits. The data concerning tests, probabilities, and costs must be manipulated in a similar fashion. The resulting testing diagram is shown in Fig. 5.

IV. DIAGNOSTIC CHART

The information given by the two testing diagrams developed for the receiver can be easily displayed in an extremely useful chart suitable for

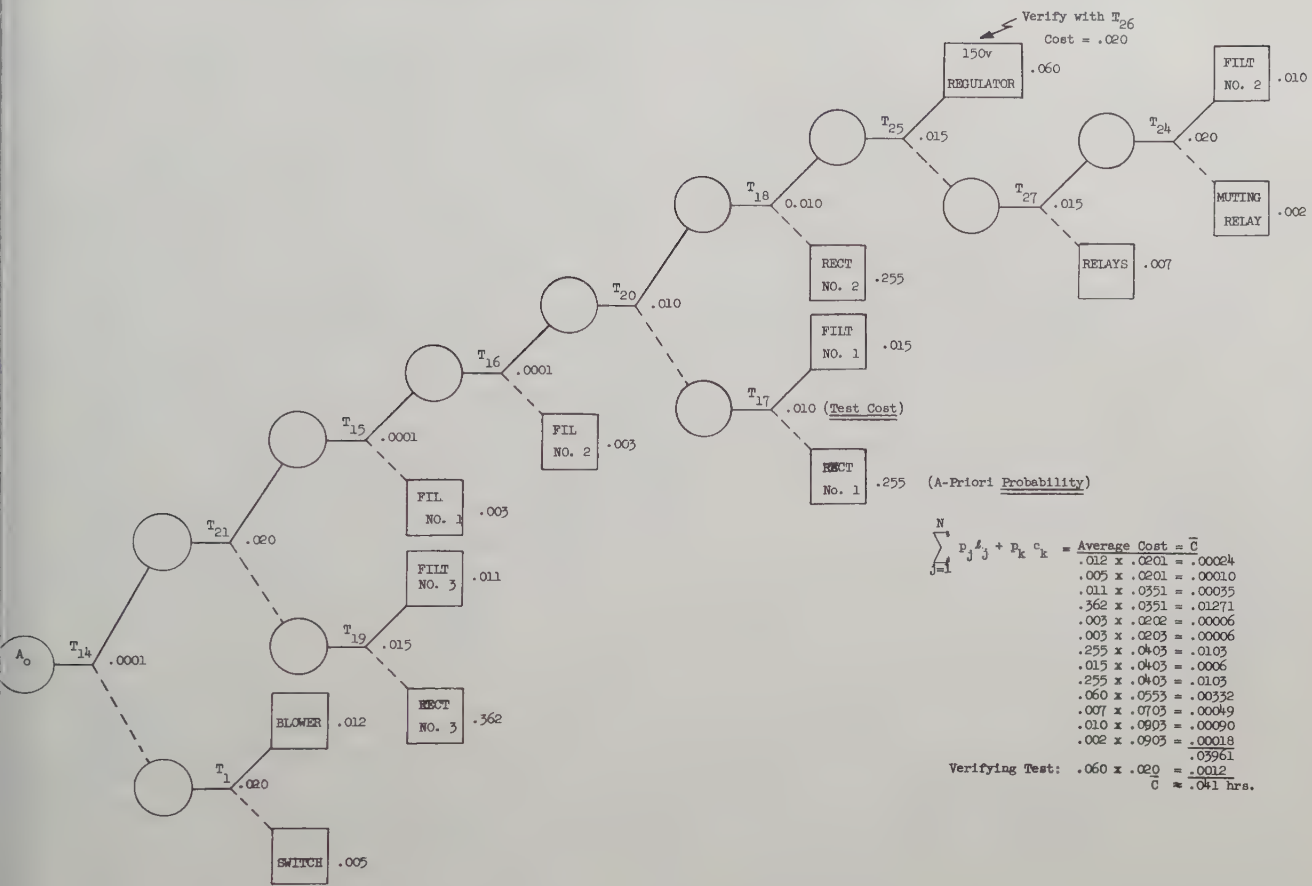

Fig. 4—Power supply testing diagram.

use by relatively inexperienced technicians. This diagnostic chart is a unique written description of the testing diagram in which the diagram is replaced by a step-by-step procedure.[5] The testing procedure to be followed at each step (test) is outlined in detail. The result of each test leads to a binary (Yes-No) result. On the basis of the result, the chart indicates the next step to be followed or the fault which has been isolated. Hence diagnosis becomes an efficient predetermined routine even for the inexperienced technician.

V. MACHINE COMPUTATION

Earlier sections have demonstrated that the information theory approach to diagnosis leads to efficient and practical fault-locating procedures.

The individual computations involved in the preparation of these procedures are simple and of a highly repetitive character. However, for large problems, hand calculation becomes exceedingly laborious and time-consuming. For this reason, a sample digital computer program has been prepared for the calculation of diagnostic procedures.[5]

The program was written specifically for the basic IBM-650 magnetic drum computer with a 2000-word drum and no external tape storage. The basic logic of this program represents one possible computing routine and can be used as starting point in writing programs for operation on larger machines and for solution of more complex problems.

The computer program is able to handle the entire computation provided that the input data fall within the following limits:

1) The number of elements, N, is $0 < N \leq 10$.
2) The number of tests, T_k, less than 100.
3) The number of unit operations with fixed cost is $0 \leq C_f \leq 25$.

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[5] E.J. Kletsky, "Diagnosis of Equipment Failures—Part ," Rome Air Dev. Center, Griffis AFB, N.Y., Tech. Rept. No. 60-67B; April, 1960.

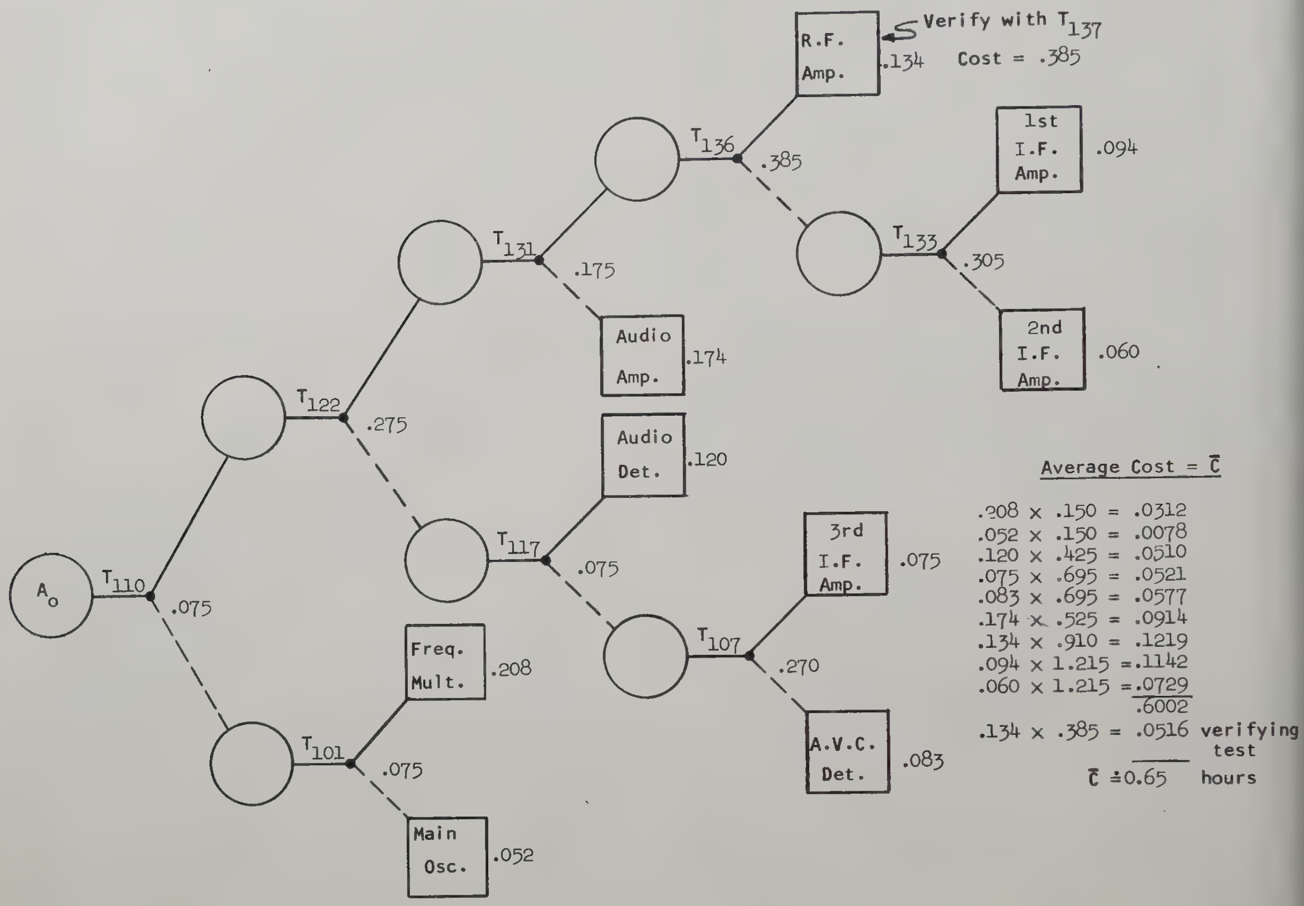

Fig. 5—Signal circuits testing diagram.

4) The number of unit operations with variable cost is $0 \leq C_V \leq 25$.
5) Each variable cost unit operation can take on an average of no more than 3 different values.
6) The maximum number of unit operations per test is 10.

This program has been used to compute the testing diagram for the signal circuits of the R-278B/GR receiver. The entire computation is carried out in approximately 4 minutes. This compares with about 3 hours for hand computation.

It is appreciated that in most practical cases, the number of tests may well exceed 100 and the number of elements will certainly exceed 10. Under these conditions, a machine with additional storage (tape, cards) becomes necessary. However, the logical structure of the program remains fixed and hence expansion for larger problems is relatively easy.

VI. GENERAL COMMENTS

Data Accumulation

Before the methods described can be applied in practice, cost data and probability data must be available. The accumulation of these data represents the most serious difficulty encountered.

The acquisition of failure probability data has been made somewhat easier by recent investigations of the RCA Service Company and by the published reliability data made available by component manufacturers. Use of these data, coupled with engineering experience, allows reasonable estimates to be made of the required functional element failure probabilities. Very accurate failure probabilities are not required, since the information gained from a test is relatively insensitive to the failure probabilities of the individual elements. This is particularly true for those tests

which yield a high information gain which are, therefore, selected by the information theory figure of merit.

Accumulation of cost data is more difficult. Little information is available concerning the cost associated with performing unit operations which go to make up the cost of a test. At present, examination of these operations in the laboratory, coupled with good engineering intuition, provides the best estimate of their costs. It is unfortunate that more precise determinations have not been made. This is particularly true in light of the fact that the figure of merit, F_K, is considerably more sensitive to variations of cost than it is to variations of probabilities. For this reason, considerable care should be given to the estimation of unit operation costs. In the development of new systems, reasonably accurate estimates of these costs can be made by personnel responsible for maintenance aspects of the design.

Generality of Procedure

The maintenance of complex electronic systems is usually carried out at several levels. These levels depend primarily on the location of the system and complicated operational and logistical problems involving allowable equipment down time, availability of test facilities and equipment, replacement inventory, manpower, and cost. The solution to the failure diagnosis problem is somewhat different for each of these levels. However, the method by which this solution is obtained remains unchanged. An example will clarify this.

A typical aircraft fire-control system shown in Fig. 6 might include a tracking radar, a computer, and a communication link with the ground based combat control center. Each of these subsystems is in turn comprised of one or more self-contained "black boxes." The black boxes perhaps contain removable subchassis, and these in turn contain the basic components (tubes, resistors, etc.) which make up the entire system.

At the first level of maintenance, the crew chief requires an efficient procedure which tells him which of the subsystems has failed so that the appropriate second-level maintenance procedure may be initiated. The first level usually consists of a sequence of tests which are performed from equipment operating positions and requiring very little external test equipment. The function of second-level maintenance is to locate and replace (or perhaps, repair) faulty black-boxes. This requires an inventory of spares but only a minimum of test equipment, tools, and time. Third- and fourth-level maintenance is usually performed at

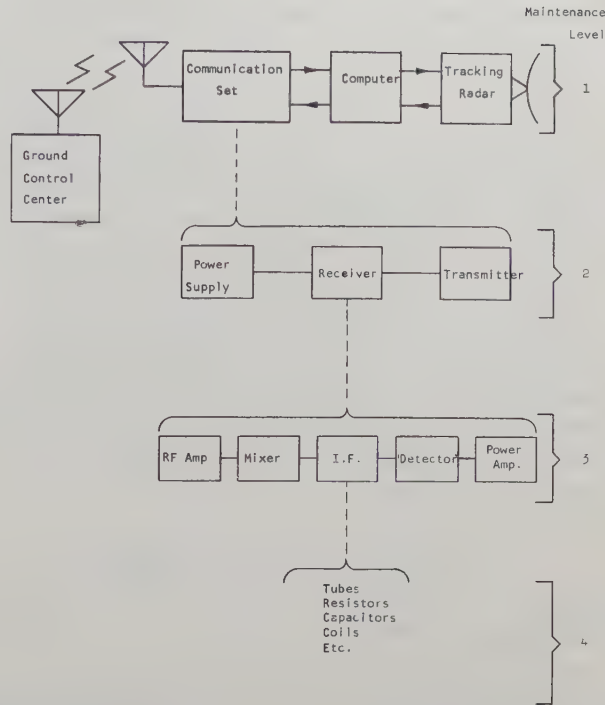

Fig. 6—Maintenance levels for hypothetical fire-control system.

a later time employing more elaborate test equipment and, in general, requiring large inventories of replacement parts.

Note that at each level of maintenance the problem is to efficiently locate the faulty functional element. It is thus seen that the fundamental difference between various maintenance levels lies in the definition of the functional units. These may vary in size from as large as an entire subsystem (radar set) to as small as a basic component (resistor or tube). The information theory approach leads to efficient diagnostic procedures for functional elements of any size.

Effect of Symptomatic Information

The sample diagnostic procedure developed for the R-278B/GR receiver assumed that there was no symptomatic information concerning the cause of failure. In general, some type of symptomatic information is always available. The operating log of the equipment indicates events prior to the failure. Discussions with the equipment operator lead to an exposition of abnormal conditions prior to and during the failure. Attempts by the operator to circumvent the failure also provide symptomatic information. A cursory visual inspection indicating

damage due to rough handling; an aural inspection indicating abnormal noises; or overheated areas are also sources of symptomatic information.

All of this symptomatic information can be incorporated in the development of efficient diagnostic procedures by noting that these symptoms effectivity alter the a priori probabilities of failure of the functional elements. For example, physical damage to the equipment case would increase the a priori probability of failure of elements such as vacuum tubes, crystals and sharply tuned circuits.

Ideally, each symptom or set of symptoms results in a different diagnostic procedure. Thus the most efficient procedure will be dictated by what symptomatic information is available. Note that absence of such information does not invalidate the general method but merely leads to procedures which may be less efficient.

VII. AREAS OF APPLICATION

Monitored Systems

The operating parameters, inputs, and outputs of many large systems are continuously monitored to provide indications of system performance. When one or more monitored signals fall outside of the prescribed limits, an alarm is set indicating a system malfunction. Interpretation of the group of out-of-tolerence signals is equivalent to performing first level maintenance. This information can be used to initiate one of a set of preprogrammed diagnostic procedures designed for second-level maintenance. Each of these preprogrammed procedures can be made efficient through the use of the information theory approach. This suggests the possibility of integrating the diagnostic procedures developed here with those of self-monitoring machines to develop systems with a self-diagnosing capability.

Check-Out Procedures

A necessary part of the maintenance problem is that of determining when the system is working. This problem occurs either following a repair action or prior to an operational mission. A check-out procedure is desired in these cases.

The diagnostic procedure developed for the equipment can be used as a check-out procedure by performing every test which lies in the upper branch of the testing diagram. If all of these tests pass, the equipment is in operating order.

Any of the diagnostic procedures determined by the information theory method can be used as a check-out procedure by performing those tests appearing in the upper branch. The optimum check-out procedure is that for which the total cost of performing the tests in the upper branch is a minimum. The testing diagram corresponding to optimum check-out is not necessarily the same as that diagram corresponding to optimum diagnosis since different criteria of optimum are used.

New Systems

Another area of applicability is that of new system design. The detailed operating characteristics, packaging proposals, and maintenance requirements of new and projected systems are obviously familiar to the personnel engaged in their design. It appears reasonable for design engineers to give strong consideration to provisions for efficient failure diagnosis during the postexperimental and prepackaging phases of new system designs. It is hoped that the techniques and concepts developed here will provide some new concepts of value to those design engineers concerned with the general problem of failure diagnosis.

Existing Systems

The techniques described can also be applied to improve the maintenance of existing operational systems. The problem of retaining military technicians over periods of time long enough for them to become efficient trouble-shooters is well known. As systems increase in complexity, this problem will become more acute.

A proposed solution is to provide each system with a set of diagnostic charts prepared in accordance with the methods described in this report. Any technician, with minimal fundamental electronic system schooling, who is able to read these charts and manipulate test equipment, should now be able to diagnosis the system nearly as well as a maintenance man with a great deal of experience with the particular system. Furthermore, the technician is not limited to maintaining a particular system, but is able to trouble-shoot any system for which a proper diagnostic chart has been prepared. The saving in training time and consequent versatility of the technicians are easily appreciated.

APPENDIX

The following is a selected sampling from among 64 tests used in preparing the testing diagrams nown in the text.

Test No.	Test of Signal	Signals to Be Supplied	UNIT OPERATIONS No.	Description	Cost	Total Initial Cost
4	C	A	28 09	Connect ac power and turn power switch ON Listen for blower Is blower operating?	0.0001 0.0000	0.0001
8	H	A	26 28 07	Remove dust cover Connect ac power and turn power switch ON Multimeter measurement at J-903 Does meter read between +100 and +125 volts dc?	0.010 0.0001 0.010	0.0201
4	N	A,Z	26 01 28 07 03	Remove dust cover Disconnect J-1206 Connect ac power and turn power switch ON Multimeter measurement at pin 1 of J-1206 Change position of channel selector switch Does meter read zero when motor is running and between +210 and +250 volts dc when motor stops?	0.010 0.005 0.0001 0.010 0.005	0.0301
6	P	A,Z	26 28 01 07 03	Remove dust cover Connect ac power and turn power switch ON Remove P-601 Multimeter measurement at pin 12 of J-1206 Change position of channel selector switch Does meter read zero when motor is running and between +148 and +152 volts dc when motor stops?	0.010 0.0001 0.005 0.010 0.005	0.0301
7	V	T'	47 10	Supply 2.05-Mc signal at J-603 VTVM measurement at AVC Jack. J-1218 Does meter begin to rise from noise level at about 0.5-volt signal and continue to rise to about - 10 volts dc for 1.5-volt signal?	0.250 0.020	0.270
)	Q	None	10 04	VTVM measurement at J-402 Run thru each of the 18 positions of the 10-Mc manual channel selector switch (from 39 to 22) Does meter read between -0.8 volt and -2 volts dc at each position?	0.020 0.055	0.075
;	W	S	27 01 42 10 06	Connect 600 ohms across output terminals (J-1213) Remove P-402 Supply 165-uv 9-Mc signal at J-505 VTVM measurement at J-1213 Run thru all 10 positions of the 0.1-Mc manual channel selector switch (from 0.9 to 0.0) Does meter read between 40 volts rms and 60 volts rms at each position?	0.100 0.005 0.250 0.020 0.030	0.405

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